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50 Years of Clos Networks A Survey of Research Issues

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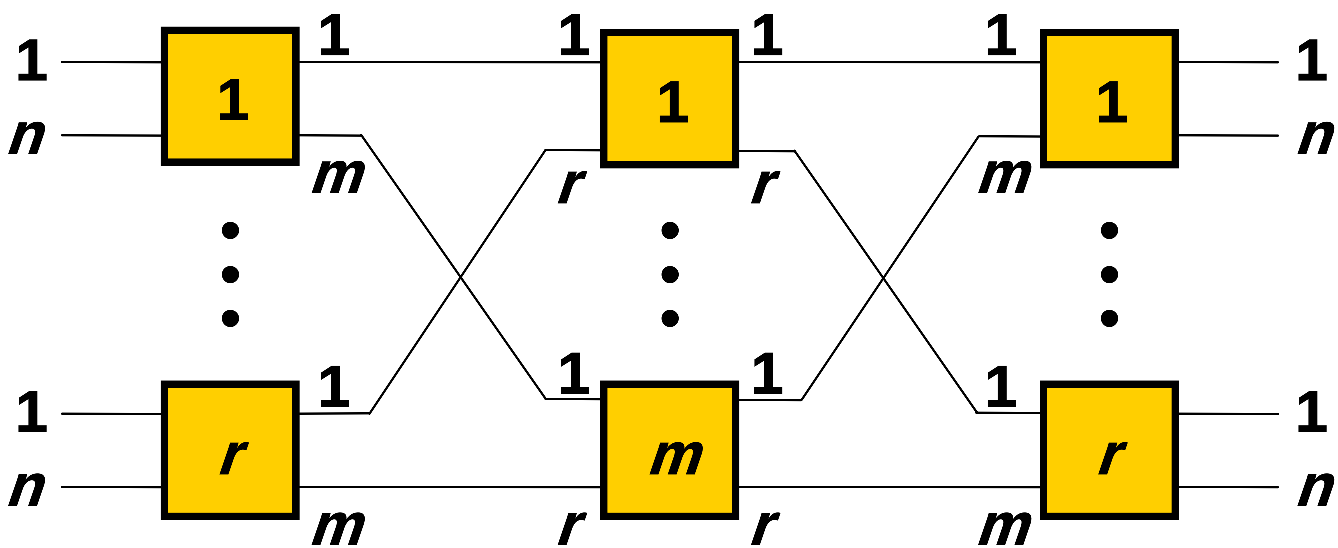
Outline

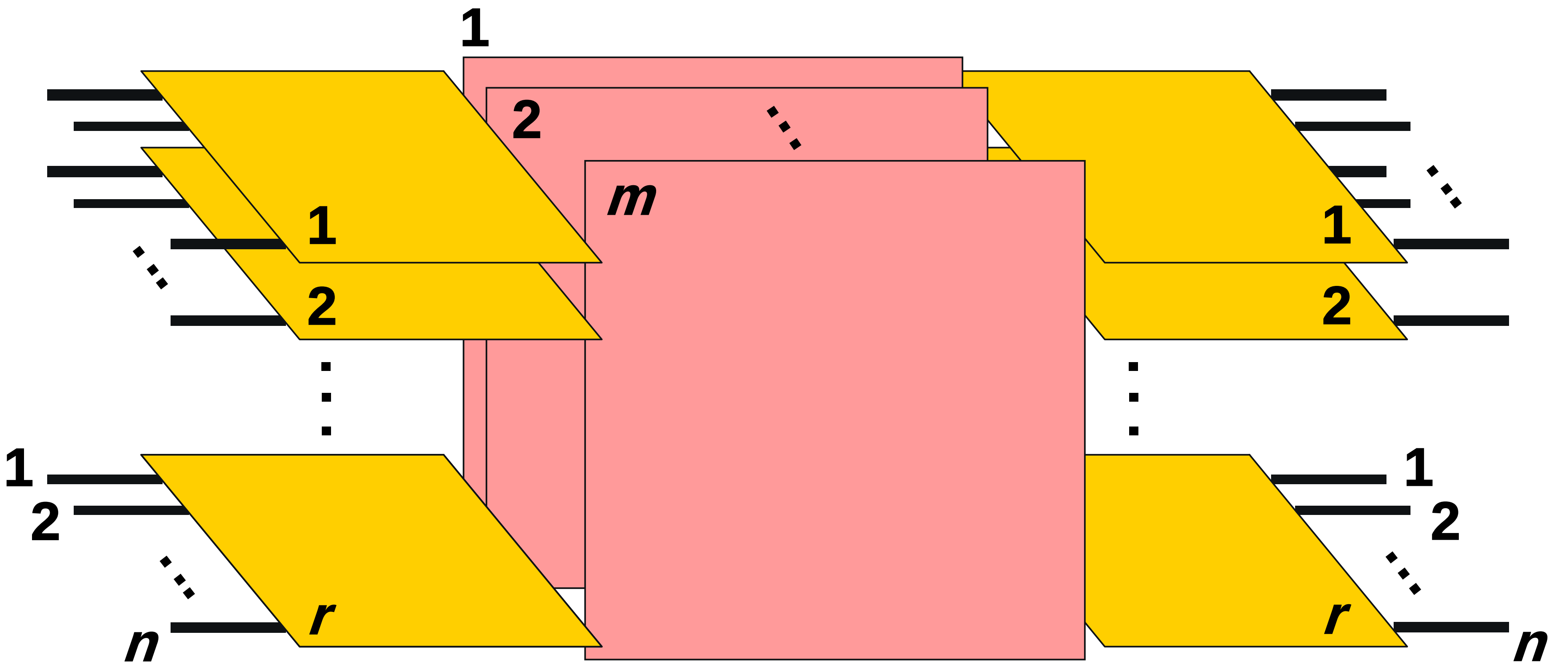
- Clos network and its variations
- Classes of Clos networks in relation to blocking
- Multicast networks
- Multirate networks
- Approaches to optimization of Clos networks
- Conclusion

Three-stage Clos network

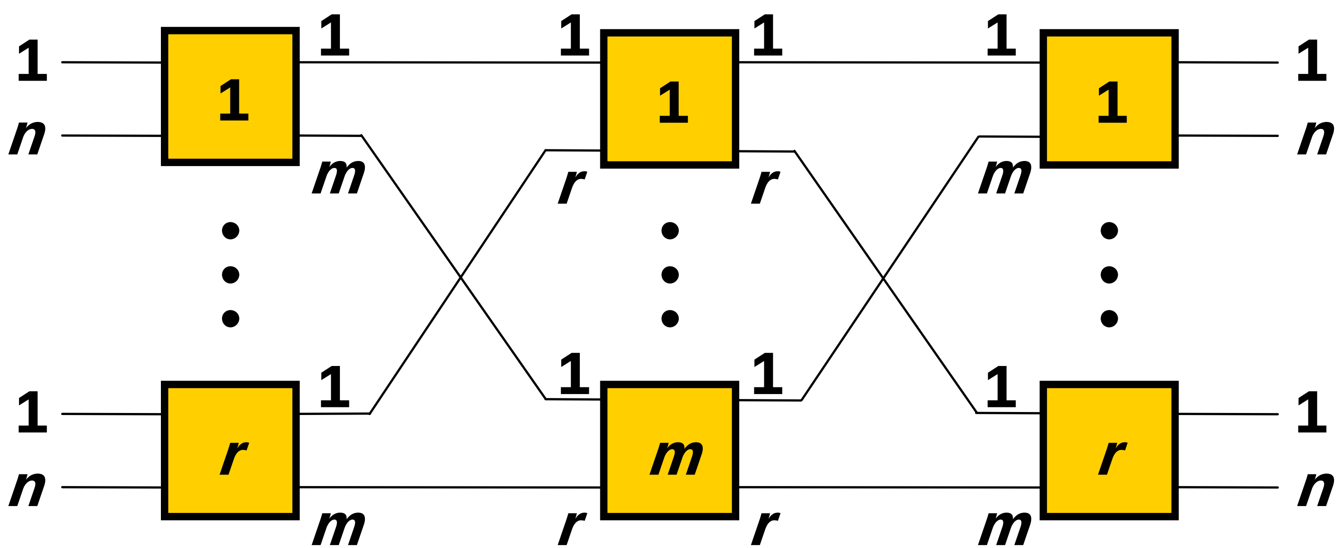


Charles Clos, "A Study of Non-blocking Switching Networks", *The Bell System Technical Journal*, 1953, vol. 32, no. 2, pp. 406-424

Alternative presentation of Clos network

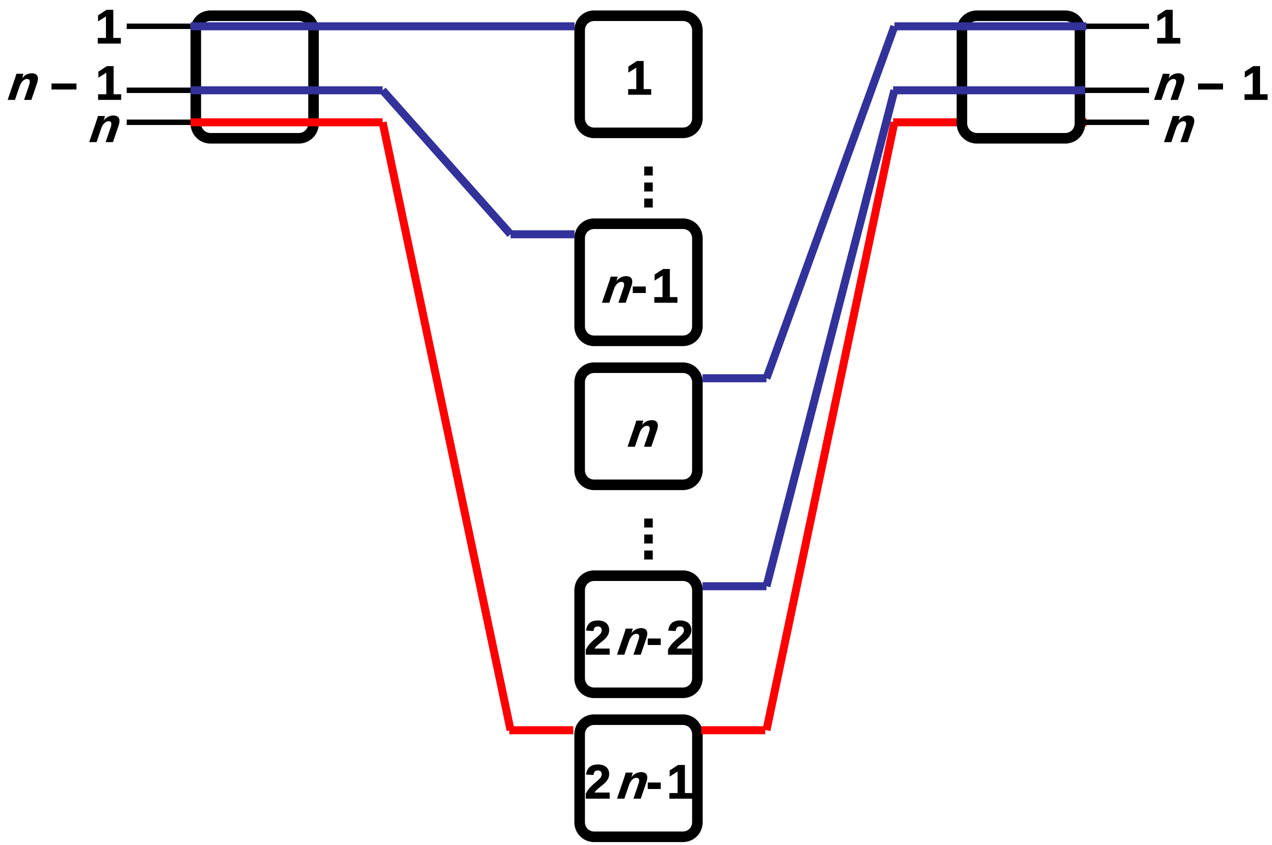


Clos theorem



Clos Theorem: **Two-sided three-stage Clos network ν(*m*,*n*,*r*) is nonblocking in the strict sense if and only if $m \geq 2n - 1$**

Proof of the Clos theorem
Sufficiency



The „worst state" of the network

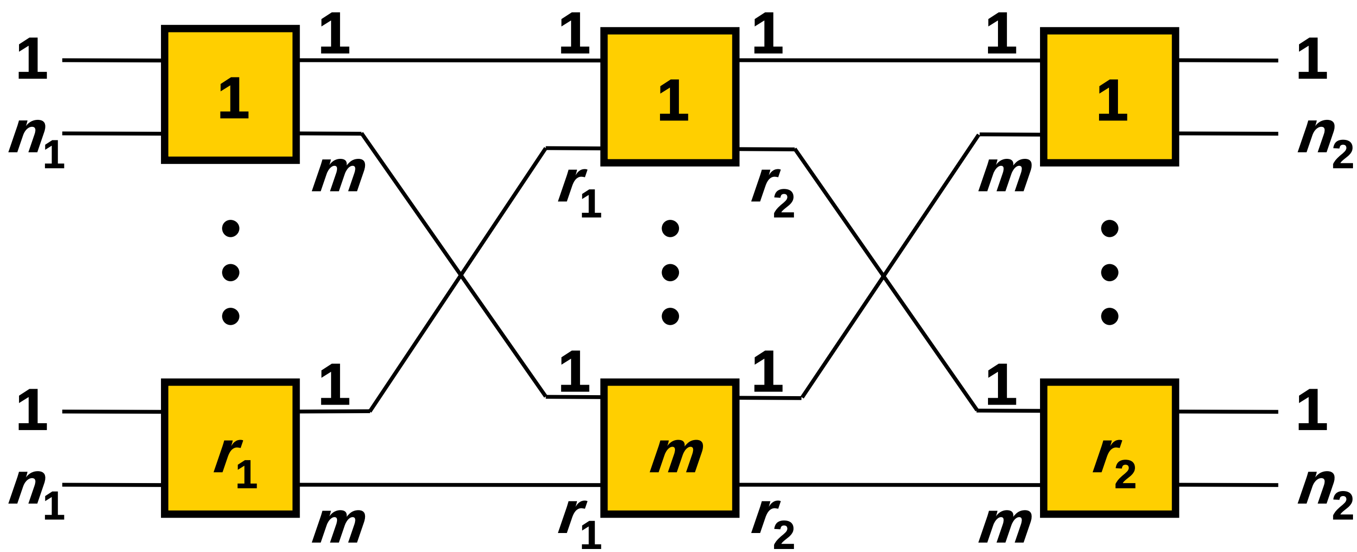
Important observation

- Clos network can be superior to a square matrix in terms of the number of required crosspoints
- Example: N = 1000

  Square matrix: C = 1000 000

  Clos network: C = 186 737

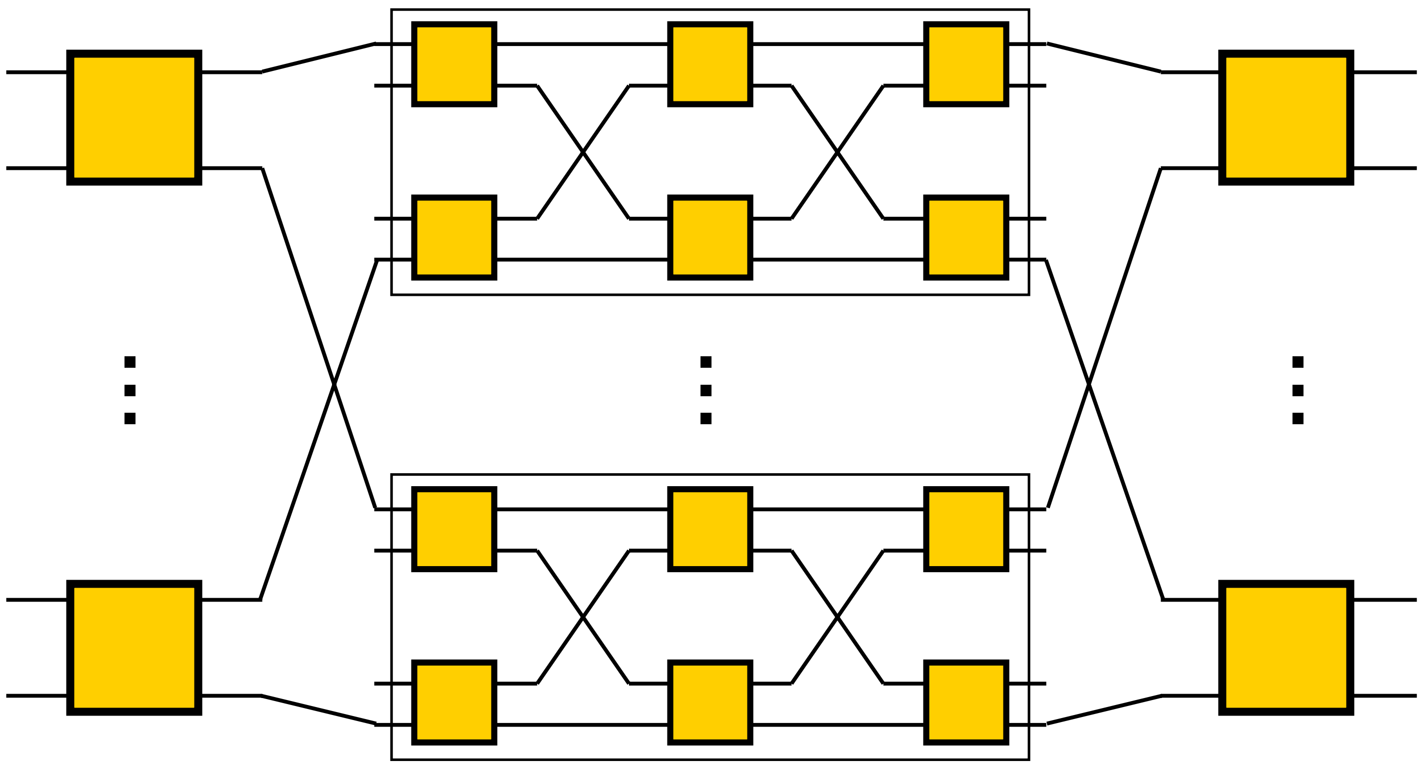
Proof of the Clos theorem
Some traps



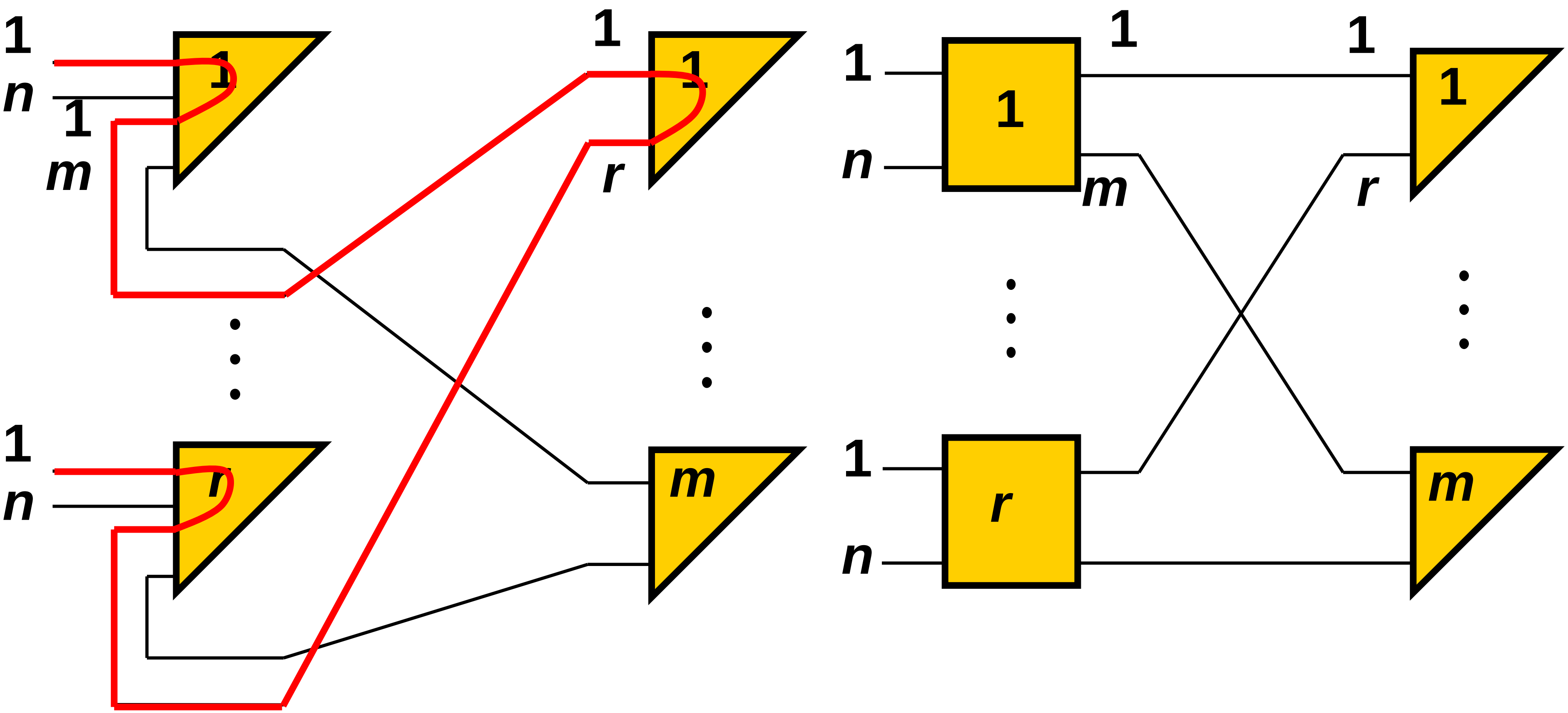
$$m \geq n_1 + n_2 - 1$$

$$m \geq \min\{n_1 + n_2 - 1, n_1 r_1, n_2 r_2\}$$

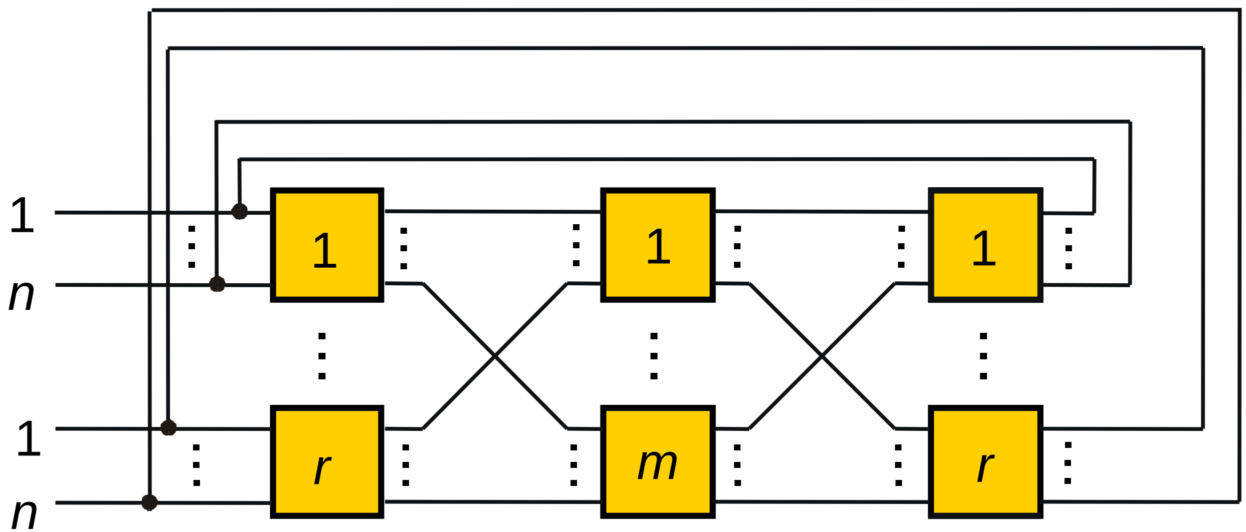
Five-stage Clos network



“Triangular” Clos networks



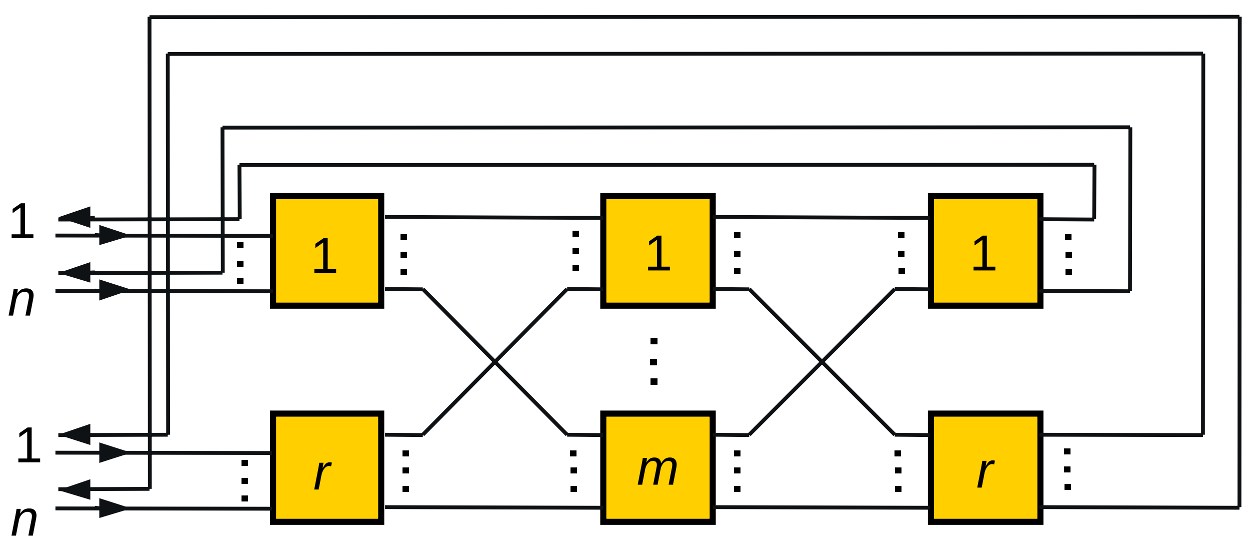
One sided networks with loops



$r > 3$ and $m \geq n$

$r \leq 3$, n is odd and $m \geq n$

$r \leq 3$, n is even and $m \geq n - 1$



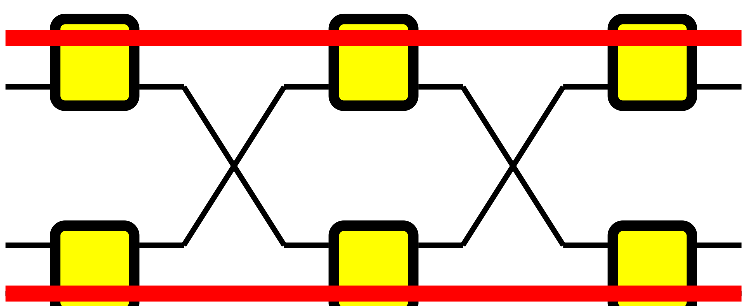
Classes of Clos networks

- **Two approaches**
  - **Functional definition**
  - **Definition in relation to blocking states**
- **Network classes**
  - **Nonblocking in the strict sense**
  - **Nonblocking in the wide sense**
  - **Repackable**
  - **Rearrangeable**
  - **Blocking**

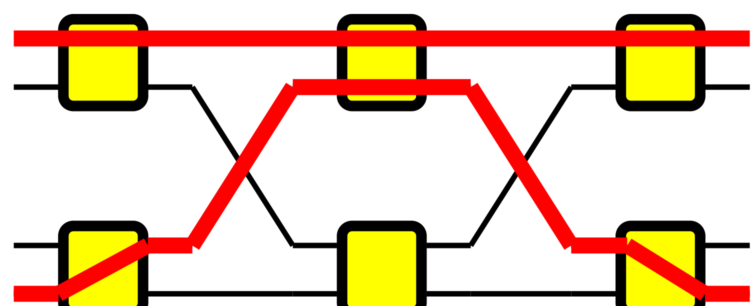
Repackable Networks

Definition 1: A ***non-permanent*** **state of a switching network is a state which when achieved by the network is immediately replaced by another state**

Definition 2: An ***overweight*** **state of a switching network is a state in which there exists a connecting path which can be disconnected and then connected again through another more heavily loaded middle-stage switch than the switch that was used before (loads are compared after the disconnection).**



An overweight state



A non-overweight state

Repackable Networks

Theorem: **No blocking occurs in a three-stage Clos network $\nu(m,n,r)$ if and only if**

$$m \geq 2n - \left\lceil \frac{n}{r-1} \right\rceil$$

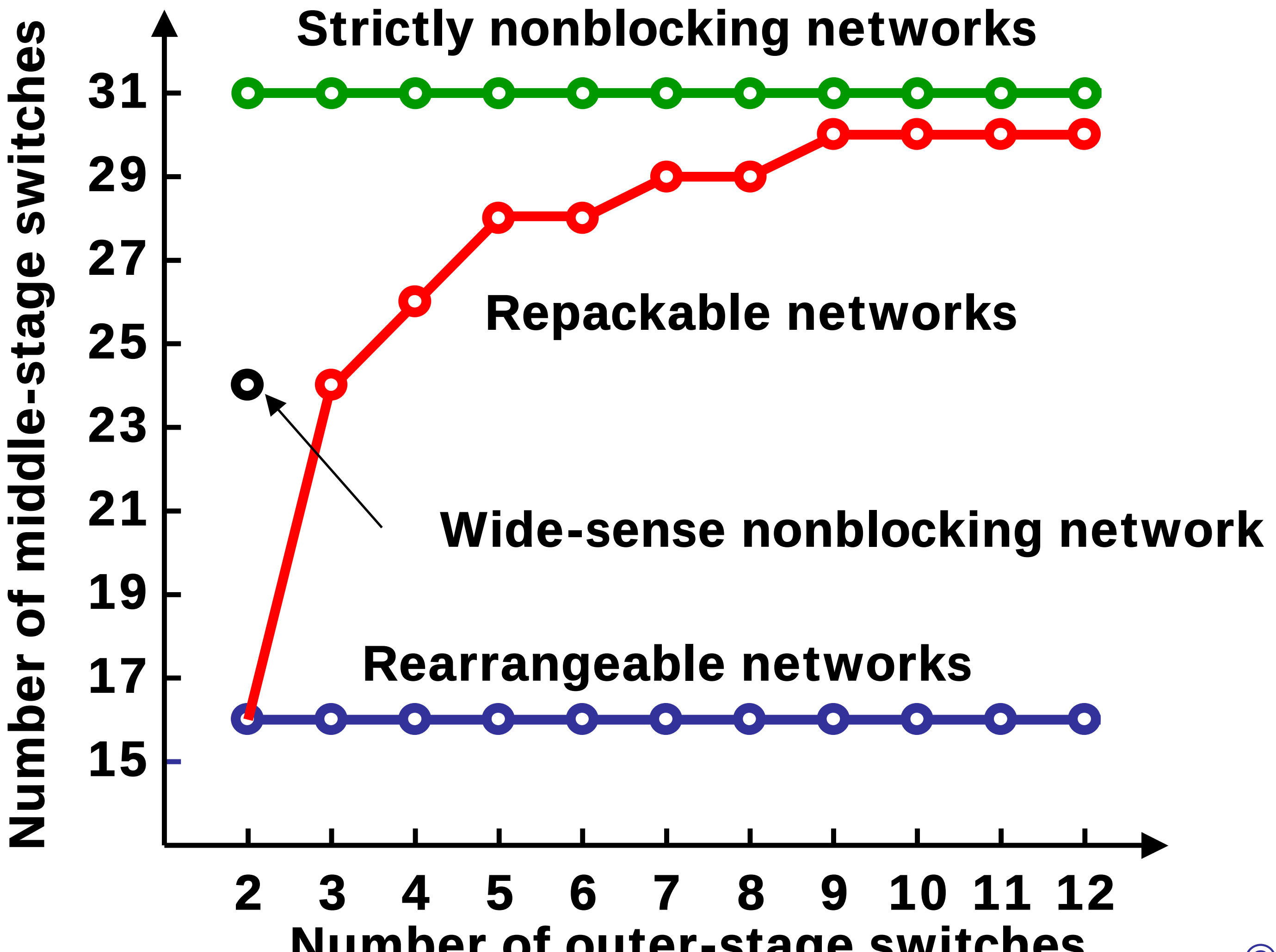
and the network is controlled in such a way that each overweight state of the network is non-permanent

***A. Jajszczyk and G. Jekel, "A New Concept — Repackable Networks", *IEEE Transactions on Communications*, vol. 41, August 1993**

Comparison of Rearrangements and Repackings

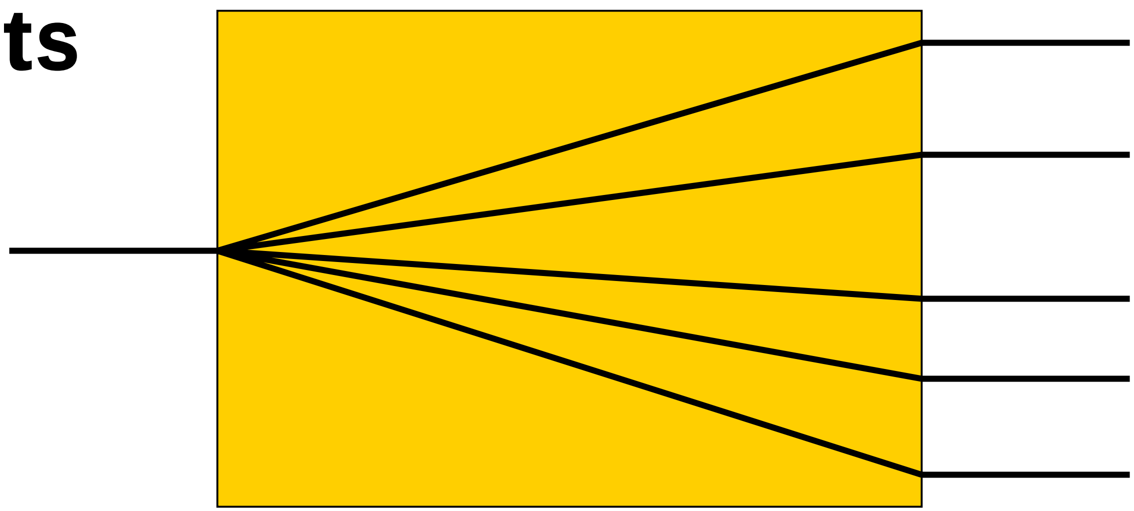
Rearrangements	Repackings
Initiated after blocking	Initiated after call termination
Strict real-time constraints	Low real-time constraints
Require centralized control	Suited to distributed control
Priority rearrangements	No priority repackings

Comparison of Clos networks
n = 16



Multicast networks

- In a multicast call, an input can request to connect to up to a certain number of outputs



- First result concerning Clos networks: G. M. Masson and B. W. Jordan, Jr., ”Generalized multi-stage connection networks”, *Networks*, vol. 2, 1972

Multicast networks
Fundamental result

- **Theorem*: A multicast strictly nonblocking network with closed-end traffic has at least $O(N^2)$ crosspoints**

*** J. Friedman, "A lower bound on strictly nonblocking network", *Combinatorica*, vol. 8, 1988, pp. 185-188**

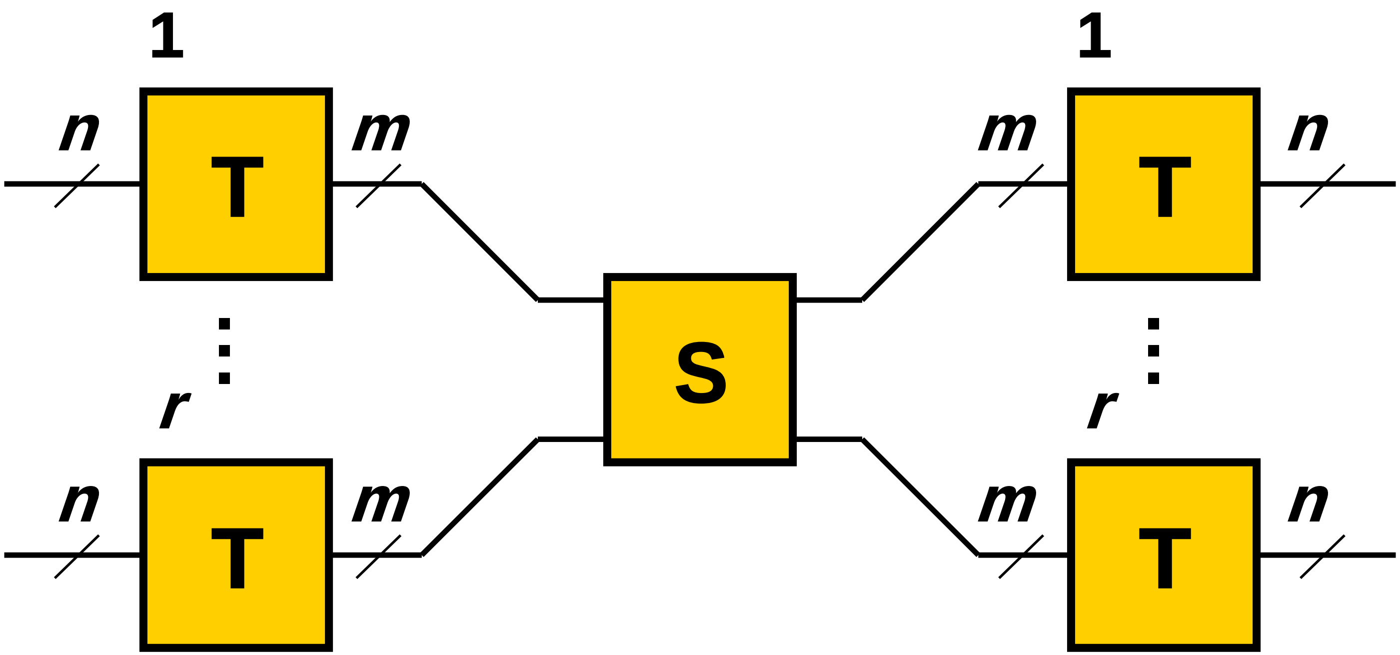
Multicast networks

- Variety of models
  - Fan-out at different stages
  - General models (with no explicit routing algorithms)

    e.g., G. M. Masson and B. W. Jordan, F. K. Hwang, A. Pattavina
  - Models with defined routing algorithms

    e.g., F. K. Hwang and A. Jajszczyk
- Good source:
  F. K. Hwang, *The Mathemetical Theory of Nonblocking Switching Networks*, World Scientific, Singapore, 1998

Time-space networks

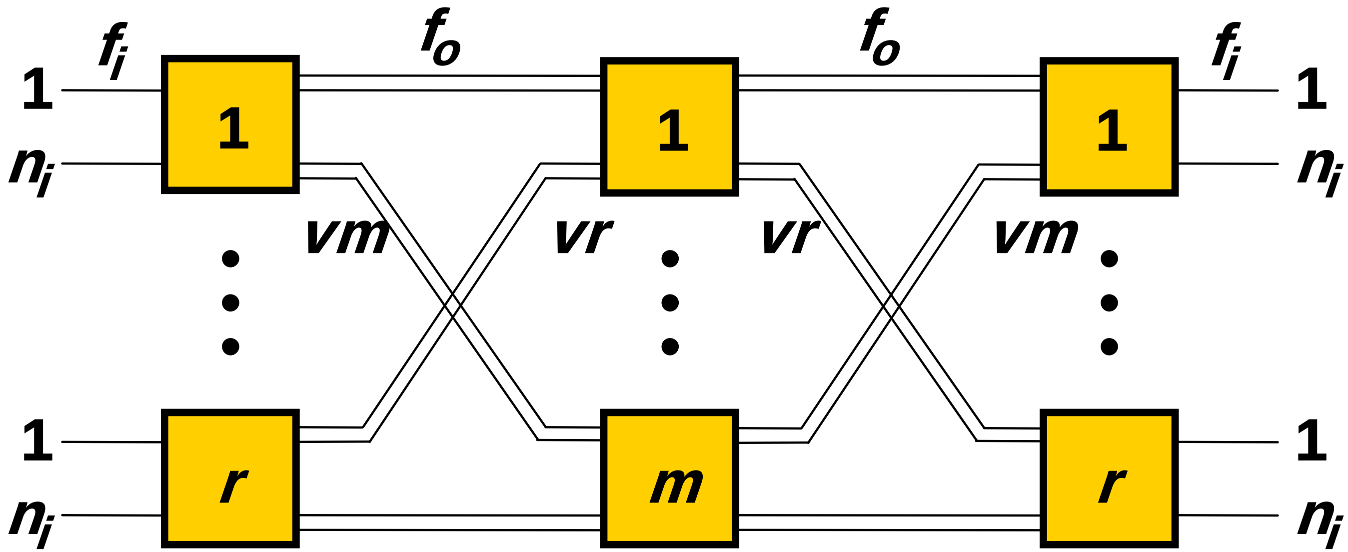


T-S-T network

Space equivalents

5 ESS: T-S-T
4 ESS: T-S-S-S-S-T
EWSD: T-S-T (T-S-S-S-T)

Conditions for DSM-based networks

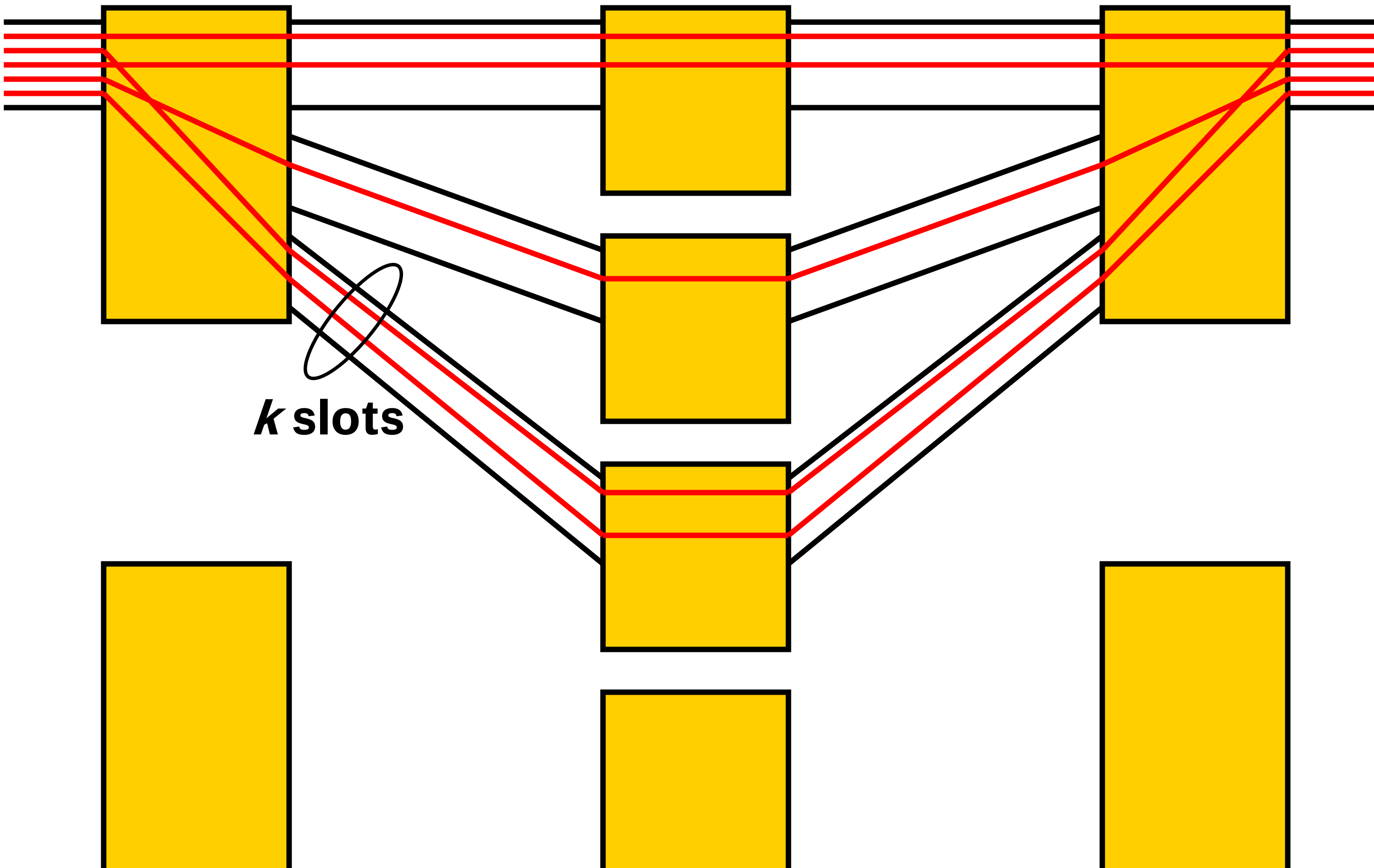


The network is nonblocking in the strict sense if and only if*:

$$m \geq 2\left\lfloor \frac{n_i f_i - 1}{v f_o} \right\rfloor + 1$$

***A. Jajszczyk, "On nonblocking switching networks composed of digital symmetrical matrices", *IEEE Transactions on Communications*, vol. COM-31, Jan. 1983**

Multi-slot connections



Multi-slot connections
Result of Niestegge*

$$m > 2\left\lfloor \frac{nk - B}{k - B + 1} \right\rfloor$$

where:

m **is the number of middle-stage switches**

n **is the number of incoming links to a first-stage switch**

k **is the number of time slots per link**

B **is the maximum number of time slots used by a single connection**

*** G. Niestegge, "Nonblocking multirate switching networks",** ***Proc. 5th ITC Seminar,*** **Lake Como, Italy, May 1987**

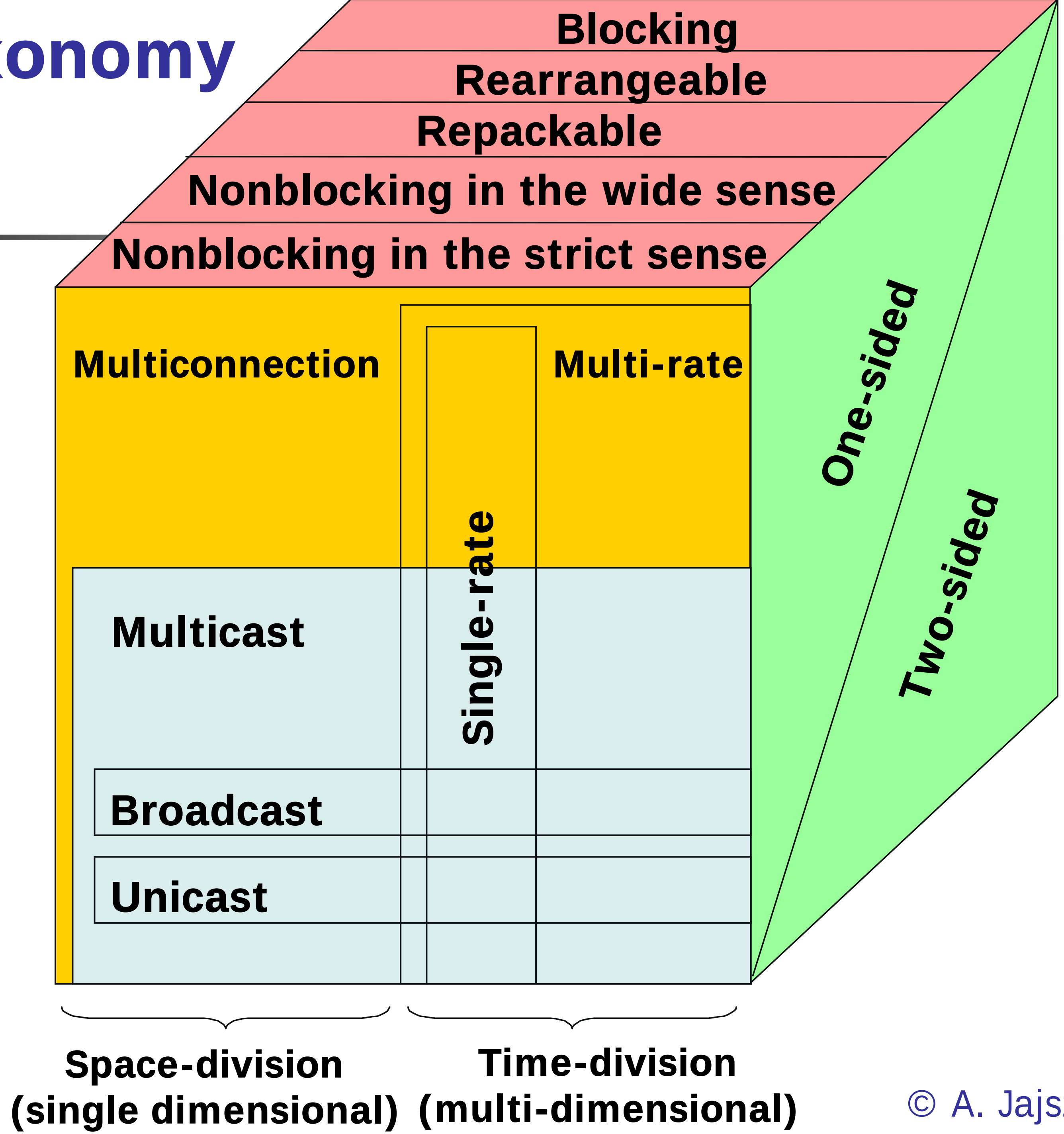
Models of multirate networks

- **Discrete model** (there is a finite number of distinct rates and the smallest rate divides all other rates)
  - **No channel grouping**
  - **Channel grouping**
- **Continuous model**
  - **Constant bit rate***

    R. Melen, J. S. Turner, "Nonblocking multirate networks",** ***SIAM J. Comput**, vol. 18, no. 2, April 1989**

    Observation: If the virtual circuit rate is approaching zero the number of middle-stage switches in strictly nonblocking networks is infinite.
  - **Varying rate traffic ("effective rate" concept)**

Taxonomy



Blocking
Rearrangeable
Repackable
Nonblocking in the wide sense
Nonblocking in the strict sense
Multiconnection
Multi-rate
Single-rate
Multicast
Broadcast
Unicast
One-sided
Two-sided
Space-division
(single dimensional)
Time-division
(multi-dimensional)

Continuous optimization of Clos networks

Optimum parameters of the strictly nonblocking Clos network $\nu(m,n,r)$

The number of crosspoints:

$$C_N = 2mnr + mr^2$$

Substituting $m = 2n - 1$, $r = N/n$, we have

$$C_N = (2n - 1)(2N + N^2/n^2)$$

The minimum of C_N is reached when $\mathrm{d}C_N/\mathrm{d}n = 0$, which gives

$$2n^3 - nN + N = 0$$

For large N (100 and more) we can approximate

$$N \approx 2n^2$$

Discrete optimization

- **Continuous optimization can lead to errors**
- **Because of the iterative structure of multistage networks, optimization can be treated as a multistage decision problem and solved by using the *dynamic programming method*. The method is based on Bellman's optimality principle that can be formulated as follows:**

***An optimal decision policy has the property that whatever the initial state of the system and the initial decision are, the rest of the decisions must form an optimal policy according to the state produced by the first decision*.**

Discrete optimization of Clos networks

In the case of Clos switching networks the problem can be reduced to the construction of tables containing optimum switching networks of *i* stages using tables of optimum (*i* – 2)-stage networks calculated earlier *.

***A. Jajszczyk, "A dynamic programming approach to optimization of switching networks composed of digital switching matrices", *IEEE Transactions on Communications*, vol. COM-35, Dec. 1987**

Measures of cost other than crosspoints

Optimum Beneš networks

Beneš network: Clos network composed of square switches

Sizes of middle-stage switches

N	*n*
2	2
3	3
2^x, $x \geq 2$	4 or 8
3^y, $y \geq 2$	3 or 9
$2^x 3^y$, $x \geq 1$, $y \geq 1$	4 or 6 or 8

The dimensions of all outer-stage switches (all except the middle-stage) have to be expressed by prime numbers or number 4

V. E. Beneš, *Mathematical Theory of Connecting Networks and Telephone Traffic*, Academic Press, New York, 1965

A. Jajszczyk, "Optimal structures of Benes switching networks", *IEEE Trans. on Commun.*, vol. 27, Feb. 1979

Conclusion

- **Clos network proved to be useful in practice and stimulating in research**
- **Unresolved theoretical problems still exist**
- **New problems arise as technology changes**
- **Feature topic devoted to 50th anniversary of Clos network: *IEEE Communications Magazine*, October 2003**