Politecnico di Torino – Dipartimento di Elettronica

Contratto CSP-Omnitel per lo sviluppo di un simulatore per reti UMTS

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On SIR & BER Approximations in DS-CDMA System

Abstract – The study presented here discusses the different approximation methods presented in the international litterature concerning the computation of the SIR (Signal to Interference Ratio) and the BER (Bit Error Rate) in DS-WCDMA systems. The content and conclusions of this document have driven the decision to implement the Standard Gaussian Approximation (SGA) in the simulator.

Since the simulator does not have a detailed physical layer simulation, the radio burst loss rate estimation will be directly based on the SGA approximation that, following the results and conclusions presented here, represent the most appropriate tradeoff between complexity and accuracy.

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Chapter 1

Introduction

Code Division Multiple Access (CDMA) is a well-known radio communication technique to allow multiple users to share the same spectrum simultaneously. In CDMA, users are multiplexed by distinct codes rather than by orthogonal frequency bands or by orthogonal time slots [1]. Direct-Sequence (DS) CDMA is the most popular of CDMA techniques. The DS-CDMA transmitter multiplies each user's signal by a distinct code waveform. The detector receives a signal composed of the sum of all users' signals, which overlap in time and frequency. In a conventional DS-CDMA system, a particular user's signal is detected by correlating the entire received signal with that user's code waveform. Multiple Access Interference (MAI) is a factor which limits the capacity and performance of DS-CDMA systems. MAI refers to the interference between direct-sequence users. This interference is the result of the random time offsets between signals, which make it impossible to design code waveforms to be completely orthogonal. While the MAI caused by any one user is generally small, as the number of interferers or their power increases, MAI becomes substantial. Therefore, any analysis of performance of a CDMA system has to take into account the amount of multiple-access interference and its effects on the parameters that measure the performance, in particular the signal-to-interference-and-noise ratio at the receiver and the related bit error probability on the information bit stream.

However, a deterministic description of the MAI is impossible to give, since the phenomenon is due to the asynchronous nature of the multiple access technique and to the propagation conditions encountered by signals emitted by different and distant sources. Thus, in system performance analysis, the presence of MAI must be taken into account by means of a proper statistical description. This solution is very attractive, because of the significant saving in computational burden that it allows in simulations; however, it can be also the most critical, because of the difficulty in determining a satisfactory and computationally efficient statistical model of the overall system.

In this report, we aim to analyze a set of solutions proposed in several works for the problem of the statistical description of the MAI in the up-link (users-to-base station link) of a DS-CDMA cellular system. The goal of our analysis is to highlight the benefits and limitations yielded by the use of those solutions, essentially by comparing them in different working conditions. It is worth to notice henceforth that the most popular approach is the Gaussian Approximation method [6] and its variants.

The work is organized as follows: in Chapter 2 we provide a mathematical model of the complete DS-

CDMA system, in Chapter 3 several analytical methods for the performance estimation are described; Chapter 4 contains the numerical results obtained from a MATLAB implementation of those methods; finally Chapter 5 provides the conclusions.

Chapter 2

System model

In this section we provide a mathematical description of an asynchronous DS-CDMA system. We focus on the up-link (or reverse link), that is the communication link from the user's terminals to the base station. We consider the possible presence of flat or frequency-selective fading and the organization of the system in a cellular structure. We describe the signal transmitted by each independent source, the channel models, the received signal and the decision statistic, in order to introduce the techniques of analysis presented in the next chapter [4, 7].

2.1 Transmitted signal

Let us assume that there are K_u independent users transmitting signals in the DS CDMA system. Each of them transmits a signal in the form:

$$s_k(t) = \sqrt{2P_k} b_k(t - \tau'_k) a_k(t - \tau'_k) \cos(\omega_c t + \theta_k) \tag{2.1}$$

where P_k is the power of the transmitted signal, ω_c is the common carrier frequency, θ_k is a phase offset, $b_k(t)$ and $a_k(t)$ are the data and spreading signals respectively and τ'_k is a random transmission delay calculated with respect to a reference transmitted signal, accounting for the lack of synchronization among the users.

The data signal $b_k(t)$ is a sequence of unit amplitude rectangular pulses of duration T_b and phase 0 or π rad with equal probability. Each pulse represents an information bit for user k. The spreading signal $a_k(t)$ is a sequence of unit amplitude rectangular pulses (chips) of duration T_c and phase 0 or π rad with equal probability. There are G_p chips per bit and thus $G_p = T_b/T_c$ is the processing gain for user k.

2.2 Channel model

The k-th source signal is transmitted through a channel $h_k(t)$ which can be represented by means of three fundamental models:

Additive White Gaussian Noise (AWGN) channel, which simply adds a white random process $n(t)$ to the delayed transmitted signal;

flat fading channel, which introduces a random path gain multiplicative factor, generally modelled with Rayleigh distribution;

frequency-selective fading channel, which generates the multipath phenomenon, that is a number of replicas of the source signal characterized by their own delays, phase rotations and Rayleigh distributed amplitudes.

The low-pass equivalent impulse response of these models can be written in the form:

$$h_k(t) = \begin{cases} e^{\jmath\phi_k}\delta(t-\tau''_k), & \text{for AWGN} \\ \alpha_k e^{\jmath\phi_k}\delta(t-\tau''_k), & \text{for flat fading} \\ \sum_{m_k=0}^{M_k}\alpha_{k,m_k}e^{\jmath\phi_{k,m_k}}\delta(t-\tau''_{k,m_k}), & \text{for frequency-selective fading} \end{cases} \tag{2.2}$$

where ϕ_k, ϕ_{k,m_k} and τ''_k, τ''_{k,m_k} are the phases and time delays introduced by the channel; they can be assumed to be random variables uniformly distributed in $[0, 2\pi)$ and $[0, T_{max}]$ respectively, where T_{max} is the maximum delay at which there can be a multipath ray. M_k is the number of multipaths generated by the frequency-selective channel for the k-th transmitted signal. α_k, α_{k,m_k} are the path gain components with Rayleigh distribution:

$$f_\alpha(\alpha) = \frac{\alpha}{\sigma_\alpha^2}\exp\left\{-\frac{\alpha^2}{\sigma_\alpha^2}\right\}. \tag{2.3}$$

2.3 Received signal

In a mobile radio CDMA system, the signals from many users arrive at the input of the receiver. Thus, the received signal contains both the desired user's signal and $K_u - 1$ undesired users' signals as well as the channel noise. In case of frequency-selective fading channels, there are also the multipath components of both the desired and interfering users. Thus, the total received signal can be written as:

$$r(t) = \sum_{k=0}^{K_u-1}\sum_{m_k=0}^{M_k-1}\sqrt{2P_k}\alpha_{k,m_k}b_k(t-\tau_{k,m_k})a_k(t-\tau_{k,m_k})\cos(\omega_c t + \phi_{k,m_k}) \ + \ n(t) \tag{2.4}$$

where $n(t)$ is the AWGN with two-sided power density $N_0/2$. Note that the value of θ_k is included here into the definition of ϕ_{k,m_k}, while the values of τ'_k and τ''_{k,m_k} are included in τ_{k,m_k}. Without loss if generality, assume that the signal from user 0 is the signal of interest.

A **correlation receiver** is typically used to filter the desired user's signal form all others users' signals which share the same bandwidth at the same time. For this purpose the received signal $r(t)$ is mixed down to baseband, multiplied by the spreading sequence associated to the desired user ($a_0(t)$) and integrated over one bit period. This sequence of operations is called *despreading*. Thus, assuming that the receiver is delay and phase synchronized with the main multipath component of the signal of interest, the bit decision statistic for that user within the bit interval $[mT_b, (m+1)T_b)$ is given by [4, 7]:

$$
\begin{aligned}
Z_0(m) &= \int_{mT_b}^{(m+1)T_b} r(t) a_0(t-\tau_{0,0}) \cos(\omega_c t) dt = \\
&= b_0(m)\alpha_{0,0}\sqrt{\frac{P_0}{2}}T_b + \sum_{k=0}^{K_u-1} \sum_{[m_k=0,\ m_0\neq 0]}^{M_k-1} I_{k,m_k} \ + \ \nu
\end{aligned}
\qquad (2.5)
$$

where $b_0(m)$ is the m-th transmitted bit from the source 0,

$$
\nu = \int_{mT_b}^{(m+1)T_b} n(t) a_0(t-\tau_{0,0}) \cos(\omega_c t) dt \qquad (2.6)
$$

is a zero-mean Gaussian random variable with variance $\sigma_\nu^2 = N_0 T_b/4$ (thermal noise component) and the summation

$$
\begin{aligned}
I &\triangleq \sum_{k=0}^{K_u-1} \sum_{[m_k=0,\ m_0\neq 0]}^{M_k-1} I_{k,m_k} = \qquad (2.7) \\
&= \sum_{k=0}^{K_u-1} \sum_{[m_k=0,\ m_0\neq 0]}^{M_k-1} \int_{mT_b}^{(m+1)T_b} \alpha_{k,m_k} s_k(t-\tau_{k,m_k}) \mathrm{e}^{\jmath\phi_{k,m_k}} a_0(t-\tau_{0,0}) \cos(\omega_c t) dt
\end{aligned}
$$

represents the contribution of MAI to the decision statistic. It is worth to notice that the MAI term includes:

- all the multipath components relative to the desired used: $I_{0,1} \dots I_{0,M_0-1}$, while the $(0,0)$-component is the direct ray;
- all the direct and multipath components relative to the interfering users: $I_{k,0} \dots I_{k,M_K-1}$ for all $k = 1, 2 \dots K_u - 1$.

Thus, the decision statistic defined in (2.5) can be re-written as:

$$
Z_0(m) = D_0(m) + I + \nu, \qquad (2.8)
$$

where $D_0(m)$ is the desired signal component (first term in (2.5)), I is the MAI (2.7) and ν is the AWGN term (2.6).

In case of cellular networks, it is worth to decompose the MAI term into two distinct contributes: $I = I_O + I_I$, where I_O is the interfering signal due to users within the same cell of the desired user (*own-cell interference*) and I_I is the interfering signal due to the presence of active users in other cells surrounding the cell of interest (*inter-cell interference*). Hence:

$$
Z_0(m) = D_0(m) + I_O + I_I + \nu, \qquad (2.9)
$$

It is conceivable to suppose that I_O and I_I are statistically independent.

A statistical description for expression (2.7) can be found in [3], [12], [4], where the term I is re-written as a function of the sets of parameters $\{\phi_{k,m_k}\}$, $\{\tau_{k,m_k}\}$, $\{P_k\}$, $\{\alpha_{k,m_k}\}$ and of the integer random variable B that represents the number of chip boundaries in the desired signal at which a transition to a different value occurs. However, the common way to proceed with the analysis disregards the complete expression of the distribution of I, by introducing instead the **Gaussian Approximation**: when the number K_u of users is large, the interfering term I (and separately the terms I_O and I_I) can be approximated by a Gaussian random variable with zero mean and variance which is the sum of the variances of the variables in the summation (Central Limit Theorem, [10], [11]). As it will be demonstrated in the chapter 3, this approximation greatly simplify the MAI analysis.

However, the Gaussian approximation — even for a large number of interfering signals — is valid only if no single user or group of users dominates the total MAI; otherwise, the Gaussian model fails [4], [5].

In case of multipath channel, a benefit from the presence of multipaths that arrive with a delay longer that one chip period (*uncorrelated multipaths*) can be obtained, by using a **RAKE receiver**.

A RAKE receiver consists of a bank of correlators, named *fingers,* each receiving a multipath signal. The RAKE receiver has a receiver finger for each resolvable multipath component. In each finger, the received signal is correlated by the spreading code, which is time-aligned with the delay of the multipath signal. After despreading, the signals are weighted and combined: when the maximal ratio combining principle is adopted, each signal is weighted by its own path gain (attenuation factor) [2]. In this case, the useful term in (2.8) becomes [7]:

$$D_0(m) = \sqrt{\frac{P_0}{2}} T_b \sum_{m_0=0}^{M_0-1} \alpha_{0,m_0} \tag{2.10}$$

recalling that M_0 indicates the number of multipath rays relative to the useful signal, received with amplitude $\sqrt{P_0}\alpha_{0,m_0}$, $\forall m_0 = 1, \dots M_0 - 1$.

2.4 Near-far effect and power control

The power control problem arises because of the multiple access interference. Due to propagation, the signal received by the base station from a user terminal close to the base station will be stronger than the signal received from another terminal located at the cell boundary. Hence, the distant users will be dominated by the close user. This is called the *near-far effect.* To achieve a considerable capacity, all signals, irrespective of distance, should arrive at the base station with the same mean power. A solution to this problem is power control, which attempts to achieve a constant received mean power for each user. Therefore, the performance of the transmitter power control is one of the several dependent factors when deciding on the capacity of a DS-CDMA system. In down-link the power control is also required, in order to minimize the interference to other cells and to compensate against the interference from other cells, but it is less critical for the performance.

2.5 Signal-to-Interference ratio

In any multiple access system, one of the fundamental design parameters is the Signal-to-Interference ratio (SIR) at the receiver, which measures the ratio between the useful power and the amount of interference generated by all the other sources sharing the same resource. Recalling expressions (2.8) and (2.9), it is easy to express the SIR:

$$SIR = \frac{E\left\{D_0(m)^2\right\}}{E\left\{(I+\nu)^2\right\}} = \frac{E\left\{D_0(m)^2\right\}}{E\left\{I_0^2\right\}+E\left\{I_I^2\right\}+E\left\{\nu^2\right\}}. \tag{2.11}$$

The statistical averages in expression (2.11) can be calculated as follows. For the useful term [3]:

$$E\left\{D_0(m)^2\right\} = E\left\{\left(b_0(m)\sqrt{\frac{P_0}{2}}\alpha_{0,0}T_b\right)^2\right\} = \frac{P_0T_b^2}{2}E\left\{\alpha_{0,0}^2\right\} \tag{2.12}$$

or — maintaining the conditioning on the path gain $\alpha_{0,0}$ —,

$$E\left\{D_0(m)^2|\alpha_{0,0}\right\} = \frac{P_0T_b^2}{2}\alpha_{0,0}^2. \tag{2.13}$$

For the noise term [3]:

$$E\left\{\nu^2\right\} = \sigma_\nu^2 = \frac{N_0T_b}{4}. \tag{2.14}$$

Expressions for the variance of the interference terms can be found in [6] [3], [4], [7], [13]; they all exploit the Gaussian approximation of the summations in I, I_O, I_I and operate subsequent statistical averages over the environmental parameters. Thus they obtain:

$$E\left\{I^2|P_k\right\} = \sigma_I^2|_{\{P_k\}} = \frac{G_pT_c^2}{6}\sum_{k=1}^{K_u-1}P_k, \tag{2.15}$$

in purely AWGN channel. In presence of flat fading:

$$E\left\{I^2|P_k,\alpha_k\right\} = \sigma_I^2|_{\{P_k,\alpha_k\}} = \frac{G_pT_c^2}{6}\sum_{k=1}^{K_u-1}\alpha_k^2P_k. \tag{2.16}$$

Finally, in case of frequency-selective fading, with the hypotheses of identical mean number M of multipaths for each source and identical mean number of users per cell, it is possible obtaining:

$$\begin{aligned} E\left\{I_O^2\right\} &= E\left\{\alpha_{k0,0}^2\right\}(K_u-1)\frac{A^2T_b^2}{3G_p} + E\left\{\alpha_{k0,m_{k0}}^2\right\}(M-1)K_u\frac{A^2T_b^2}{3G_p} = \\ &= \frac{A^2T_b^2 2\sigma^2(MK-1)}{3G_p} \end{aligned} \tag{2.17}$$

$$\begin{aligned} E\left\{I_I^2\right\} &= E\left\{\alpha_{k1,m_{k1}}^2\right\}E\left\{\beta_{k1}^4\right\}MK\frac{A^2T_b^2}{3G_p} = \\ &= 2\sigma^2\frac{MK}{5}\frac{A^2T_b^2}{3G_p} \end{aligned} \tag{2.18}$$

where $\alpha_{k_0,m_{k0}}$ are the path gains affecting signals of the reference cell, $\alpha_{k_1,m_{k1}}$ are the path gains affecting signals of the surrounding cells, $\beta_{k_1} = \frac{r_{1,k1}}{r_{0,k0}}$ is the ratio between the distances of the k_1-th user of a surrounding cell from its home base station ($r_{1,k1}$) and from the reference base station ($r_{0,k0}$). If the path gains are identically Rayleigh distributed and β_{k_1} is uniform in $(0, 1]$, we obtain:

$$E\left\{\alpha_{k_0,0}^2\right\} = E\left\{\alpha_{k_0,m_{k0}}^2\right\} = E\left\{\alpha_{k_1,m_{k1}}^2\right\} = 2\sigma^2 \tag{2.19}$$

and

$$E\left\{\beta_{k_1}^4\right\} = \frac{1}{5}. \tag{2.20}$$

Chapter 3

Techniques of analysis

3.1 Standard Gaussian Approximation (SGA)

The use of the Gaussian Approximation to determine the Signal-to-Interference Ratio (SIR) and the Bit Error Rate (BER) for a CDMA communications system is based on the argument that the bit decision statistic Z_0 (2.8) may be modelled as a Gaussian random variable [8], [9].

We recall here the expression of the decision statistic of the transmitted bit as derived in the previous chapter:

$$Z_0(m) \quad = \quad D_0(m) + I + \nu \tag{3.1}$$

where D_0 is the useful information component and represents a deterministic variable given the transmitted bit, while the MAI and thermal noise components, I and ν, are independent zero-mean Gaussian random variables. Thus, defining $\xi = I + \nu$, Z_0 is a Gaussian random variable with mean D_0 and a variance which is equal to the variance of ξ (σ_ξ^2).

3.1.1 Propagation in absence of fading

Thanks to the statistical independence of the thermal noise and MAI terms, the variance σ_ξ^2 is directly expressed as:

$$\begin{aligned} \sigma_\xi^2 &= \sigma_I^2 + \sigma_\nu^2 = \\ &= \frac{G_p T_c^2}{6} \sum_{k=1}^{K_u - 1} P_k + \frac{N_0 T_b}{4} \end{aligned} \tag{3.2}$$

Then, because of the Gaussian distribution of the noise+interference term ξ, the probability of a bit error over the channel is given by:

$$BER = Q\left(\frac{|D_0|}{\sigma_\xi^2}\right) = Q\left(\sqrt{\frac{P_0 T_b^2}{2\sigma_\xi^2}}\right). \tag{3.3}$$

Now consider that, for QPSK and BPSK modulation schemes, the relation between the bit error probability and the signal-to-noise ratio (E_b/N_0) over Additive White Gaussian Noise (AWGN) channel in absence of interferers is expressed by the well-known relation:

$$BER = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) \tag{3.4}$$

where E_b is the energy per bit and $N_0/2$ is the two-sided power spectral density of the thermal noise. Therefore, the previous expression yields the definition of an equivalent signal-to-interference-and-noise ratio (SIR) for the CDMA system: by comparing equations (3.3) and (3.2), the following expression is straightforward obtained:

$$SIR = \frac{P_0 T_b^2}{2\sigma_\xi^2} = \tag{3.5}$$

$$= \frac{0.5}{\frac{1}{3G_p}\sum_{k=1}^{K_u-1}\frac{P_k}{P_0} + \frac{N_0}{2T_b P_0}} \tag{3.6}$$

In typical mobile radio environments, communication links are interference-limited and not noise-limited. For the interference-limited case the thermal noise term can be neglected and the average SIR and the average BER are given by

$$SIR|_{\{P_k\}} = \frac{3G_p}{2\sum_{k=1}^{K_u-1}\frac{P_k}{P_0}} \tag{3.7}$$

$$BER|_{\{P_k\}} = Q\left(\sqrt{\frac{3G_p}{\sum_{k=1}^{K_u-1}\frac{P_k}{P_0}}}\right) \tag{3.8}$$

Note that the previous expressions assume the knowledge of the set of the received powers $\{P_k\}$.

CDMA systems generally implement some form of power control, in order to reduce the near-far effect. Thus, ideally, all the signals arrive at the receiver with the same power: $P_k = P_0, \forall k$. In this case:

$$SIR = \frac{0.5}{\frac{K_u-1}{3G_p} + \frac{N_0}{2T_b P_0}} \tag{3.9}$$

$$BER = Q\left(\sqrt{\frac{1}{\frac{K_u-1}{3G_p} + \frac{N_0}{2T_b P_0}}}\right) \tag{3.10}$$

Finally, in the interference-limited case with perfect power control, the average SIR and the average BER can be approximated by

$$SIR = \frac{3G_p}{2(K_u-1)} \tag{3.11}$$

$$BER = Q\left(\sqrt{\frac{3G_p}{K_u-1}}\right) \tag{3.12}$$

3.1.2 Fading Channels

If we consider a frequency non-selective fading channel, the path gain component α_k can be modelled as a Rayleigh random variable with distribution (2.3). In this case the SIR and the BER, conditioned on the set of fading amplitudes and received powers, are given by [4]:

$$SIR|_{\{\alpha_k, P_k\}} = \frac{1}{\frac{1}{3G_p}\sum_{k=1}^{K_u-1}\frac{P_k}{P_0}\frac{\alpha_k^2}{\alpha_0^2} + \frac{N_0}{2T_b P_0 \alpha_0^2}} \quad (3.13)$$

$$BER|_{\{\alpha_k, P_k\}} = Q\left(\sqrt{\frac{1}{\frac{1}{3G_p}\sum_{k=1}^{K_u-1}\frac{P_k}{P_0}\frac{\alpha_k^2}{\alpha_0^2} + \frac{N_0}{2T_b P_0 \alpha_0^2}}}\right) \quad (3.14)$$

At last, the propagation channel can be modelled as a frequency-selective channel with fourth-order propagation path loss and impulse response generating M multipaths per signal, each of them independently faded with Rayleigh statistics. In this case, the RAKE receiver structure can combat the frequency selection effect by usefully exploiting the presence of uncorrelated multipath rays. Proper SIR and BER expressions for the case of a RAKE structure are presented in the next section.

3.1.3 Cellular scenario

In case of cellular system, the decision statistic Z_0 can be written as (2.9):

$$Z_0(m) = b_0(m) A \alpha_{0,0} T_b + I_O + I_I + \nu \quad (3.15)$$

where $A = \sqrt{2P_0}$ is the received desired signal amplitude, I_O is the own-cell interference and I_I is the inter-cell interference. Again, assuming the independence and Gaussian distribution for I_O and I_I, the average SIR and the average BER, conditioned on the knowledge of $\alpha_{0,0}$, can be approximated as [7]:

$$SIR|_{\alpha_{0,0}} = \frac{(A\alpha_{0,0}T_b)^2}{E\left\{I_O^2\right\} + E\left\{I_I^2\right\} + E\left\{\nu^2\right\}}. \quad (3.16)$$

In the case of N_c interfering cells equipped by a conventional correlation-type receiver at the base station and perfectly implementing the power control, the SIR (3.16) and the BER can be calculated by using expressions (2.17), (2.18), (2.14):

$$SIR_M|_{\alpha_{0,0}} = \frac{\alpha_{0,0}^2}{\frac{N_0}{2E_b} + \frac{2\sigma^2}{3G_p}\left[\left(1+\frac{N_c}{5}\right)MK_u - 1\right]} \quad (3.17)$$

$$BER_M|_{\alpha_{0,0}} = Q\left(\sqrt{SIR_M}\right) = Q\left(\sqrt{\frac{\alpha_{0,0}^2}{\frac{N_0}{2E_b} + \frac{2\sigma^2}{3G_p}\left[\left(1+\frac{N_c}{5}\right)MK_u - 1\right]}}\right) \quad (3.18)$$

where σ^2 is the variance of the Rayleigh random variable modelling the fading process. Then, averaging over the distribution of $\alpha_{0,0}$:

$$SIR_M = \frac{2\sigma^2}{\frac{N_0}{2E_b} + \frac{2\sigma^2}{3G_p}\left[\left(1+\frac{N_c}{5}\right)MK_u - 1\right]} \tag{3.19}$$

$$BER_M = \frac{1}{2}\left[1 - \frac{1}{\sqrt{1 + \frac{N_0}{2E_b\sigma^2} + \frac{2}{3G_p}\left[\left(1+\frac{N_c}{5}\right)MK_u - 1\right]}}\right] \tag{3.20}$$

where the $Q(.)$ function disappeared in the solution of the integral, thanks to the particular distribution of $\alpha_{0,0}$ which has been assumed a Rayleigh random variable.

If a RAKE receiver is used, all the uncorrelated multipath components contribute to the useful signal. So, with perfect power control, the SIR and the BER can be derived as [7]:

$$SIR_{RM}|_x = \frac{x}{\frac{N_0}{2E_b} + \frac{2\sigma^2}{3G_p}\left[\left(1+\frac{N_c}{5}\right)MK_u - 1\right]} \tag{3.21}$$

$$BER_{RM}|_x = Q\left(\sqrt{\frac{x}{\frac{N_0}{2E_b} + \frac{2\sigma^2}{3G_p}\left[\left(1+\frac{N_c}{5}\right)MK_u - 1\right]}}\right) \tag{3.22}$$

where

$$x = \sum_{m_1=1}^{M} \alpha_{m_1 i}^2 \tag{3.23}$$

is a χ^2 random variable with the following distribution:

$$f(x) = \frac{x^{M-1}e^{-x/2\sigma^2}}{(2\sigma^2)^M (M-1)!}. \tag{3.24}$$

Averaging expressions (3.21) and (3.22) over the distribution (3.23), it is possible to obtain:

$$SIR_{RM} = \frac{2M\sigma^2}{\frac{N_0}{2E_b} + \frac{2\sigma^2}{3G_p}\left[\left(1+\frac{N_c}{5}\right)MK_u - 1\right]} \tag{3.25}$$

$$BER_{RM} = \int_0^{+\infty} Q\left(\sqrt{\frac{x}{\frac{N_0}{2E_b} + \frac{2\sigma^2}{3G_p}\left[\left(1+\frac{N_c}{5}\right)MK_u - 1\right]}}\right) f(x)\,dx \tag{3.26}$$

3.1.4 Imperfect power control

Up to this point, the presence of an imperfect power control has been handled in a deterministic manner. However, in order to obtain averaged results on many possible working conditions, a statistical description of the imperfect power control phenomenon has to be given.

When the power control is imperfect, the received amplitude A_k of the k-th user can be modelled as random variable with *uniform distribution* around the nominal value of the received power level A_0. This means that the probability density function of A_k can be assumed as

$$f(A_k) = \frac{1}{2V} \qquad A_0 - V \leq A_k \leq A_0 + V \tag{3.27}$$

where V is the maximum variation range of the received signal with respect to the mean value A_0. Then, in the case of conventional correlation-type receiver, the average SIR and the average BER are given by [7]:

$$SIR_{IM} = \frac{2\sigma^2}{\frac{N_0}{2E_b} + \frac{2\sigma^2}{3G_p}\left(1 + \frac{V^2}{3A_0^2}\right)\left[\left(1 + \frac{N_c}{5}\right)MK_u - 1\right]} \tag{3.28}$$

$$BER_{IM} = \frac{1}{2}\left[1 - \frac{1}{\sqrt{1 + \frac{N_0}{2E_b\sigma^2} + \frac{2}{3G_p}\left(1 + \frac{V^2}{3A_0^2}\right)\left[\left(1 + \frac{N_c}{5}\right)MK_u - 1\right]}}\right] \tag{3.29}$$

As in the previous case, the $Q(.)$ function disappeared from the BER expression thanks to the solution of the integral for the Rayleigh distribution.

Instead, with a RAKE receiver, the SIR and the BER are given by

$$SIR_{IRM}|_x = \frac{x}{\frac{N_0}{2E_b} + \frac{2\sigma^2}{3G_p}\left(1 + \frac{V^2}{3A_0^2}\right)\left[\left(1 + \frac{N_c}{5}\right)MK_u - 1\right]} \tag{3.30}$$

$$BER_{IRM}|_x = Q\left(\sqrt{\frac{x}{\frac{N_0}{2E_b} + \frac{2\sigma^2}{3G_p}\left(1 + \frac{V^2}{3A_0^2}\right)\left[\left(1 + \frac{N_c}{5}\right)MK_u - 1\right]}}\right) \tag{3.31}$$

Finally, in absence of fading and interfering cells, the previous expressions (3.28) and (3.29) can be simplified in:

$$SIR_{IM} = \frac{1}{\frac{N_0}{2E_b} + \frac{K_u - 1}{3G_p}\left(1 + \frac{V^2}{3A_0^2}\right)} \tag{3.32}$$

$$BER_{IM} = Q\left(\sqrt{SIR_{IM}}\right) \tag{3.33}$$

and in analogous way for (3.30) and (3.31).

Note that the uniform distribution for A_k is a rough approximation: better distribution model could be selected, that more carefully takes into account the impairments of the practical realization of the power control algorithm (e.g. the "quantization" of the possible transmitted levels).

3.1.5 Absence of power control

In some situations the base station does not implement any form of power control on the users' channels. In those cases, the received powers are strongly dependent on the distance between the transmitting terminals and the base station and the near-far problem strongly arises. In particular, assuming a constant transmitted power, the received power level is attenuated by a path loss factor that is proportional to r^n, being r the transmitter-receiver distance and n the path-loss exponent ranging from $n = 2$ for free-space propagation to $n = 4$ for urban environments [3]. Moreover, the received power is still subject to the Rayleigh fading phenomenon.

In order to take into account the distance-dependent path-loss, we assume a uniform distribution of the users within a circular cell of radius R_c. Assuming that r_o is the minimum distance between the base station antenna and the user terminal, the statistical distribution of the distance r is:

$$f_r(r) = \frac{2r}{R_c^2 - r_o^2}, \qquad r \in [r_o, R_c]. \tag{3.34}$$

The received signal amplitude A at the base station is a function of the distance r, of the path-loss exponent n and of the transmitted amplitude A_0:

$$A = \frac{2A_0}{(\sqrt{r})^n}. \tag{3.35}$$

Therefore its distribution $f_A(a)$ can be derived as follows: let us consider the cumulative distribution $F_A(a)$ of the amplitudes; it is defined as the probability that A is lower than a:

$$F_A(a) = Pr\{A < a\} = \int_{-\infty}^{a} f_A(a)\, da. \tag{3.36}$$

This probability can be evaluated by means of (3.35) as:

$$F_A(a) = Pr\{A < a\} = Pr\left\{\frac{A_0}{r^{n/2}} < a\right\} = \tag{3.37}$$

$$= 1 - Pr\left\{r \leq \left(\frac{A_0}{a}\right)^{2/n}\right\}, \tag{3.38}$$

i.e., by putting $\tilde{r} = \left(\frac{A_0}{a}\right)^{2/n}$,

$$F_A(a) = \begin{cases} 0, & \tilde{r} > R_c, \quad a < \frac{A_0}{R_c^{n/2}} \\ 1 - \int_{r_o}^{\tilde{r}} f_r(r)dr, & \tilde{r} \in (0, R_c], \quad \frac{A_0}{R_c^{n/2}} < a < \frac{A_0}{r_o^{n/2}} \\ 1, & r < r_o, \quad a > \frac{A_0}{r_o^{n/2}}. \end{cases} \tag{3.39}$$

Then, by considering that $f_A(a) = \frac{d}{da}F_A(a)$, it is easy to obtain:

$$f_A(a) = \begin{cases} \frac{4A_0^{4/n}}{n(R_c^2 - r_o^2)} a^{-\frac{4+n}{n}}, & \frac{A_0}{R_c^{n/2}} < a < \frac{A_0}{r_o^{n/2}} \\ 0, & otherwise. \end{cases} \tag{3.40}$$

At this point the mean value μ_A and the variance σ_A^2 of the received signal amplitudes in absence of power control can be calculated:

$$\mu_A = E\{A\} = \begin{cases} \frac{4A_0}{(n-4)(R_c^2-r_o^2)}\left[r_o^{\frac{4-n}{2}} - R_c^{\frac{4-n}{2}}\right], & n \neq 4 \\ \frac{A_0}{(R_c^2-r_o^2)}\left[\ln\left(\frac{A_0}{r_o^{n/2}}\right) - \ln\left(\frac{A_0}{R_c^{n/2}}\right)\right], & n = 4. \end{cases} \tag{3.41}$$

$$\sigma_A^2 = E\left\{A^2\right\} - \mu_A^2 = \frac{A_0^2}{(n-1)(R_c^2 - r_o^2)}\left[r_o^{2-n} - R_c^{2-n}\right] - \mu_A^2. \tag{3.42}$$

It is worth to notice that the previous expressions can be easily extended to the case of interference from other cells: if we consider a circular region of interfering cells around that of interest, with uniformly distributed users, their interfering amplitudes are still statistically described by (3.41) and (3.42) by substituting R_c with the maximum possible distance of a user from the base station of interest and r_o with R_c.

Now, following the development which leads to (3.28) and (3.29) [7], the SIR and BER expressions in absence of power control become:

$$SIR_{AM} = \frac{2\sigma^2}{\frac{2\sigma^2}{3G_p}\frac{1}{\mu_A^2}\left[E\left\{A_O^2\right\}(MK_u - 1) + \frac{N_c M K_u}{5}E\left\{A_I^2\right\}\right] + \frac{N_0}{2E_b}} \tag{3.43}$$

$$BER_{AM} = \frac{1}{2}\left[1 - \frac{1}{\sqrt{1 + \frac{N_0}{2E_b\sigma^2} + \frac{2}{3G_p}\frac{1}{\mu_A^2}\left[E\left\{A_O^2\right\}(MK_u - 1) + \frac{N_c M K_u}{5}E\left\{A_I^2\right\}\right]}}\right] \tag{3.44}$$

where A_O is the random variable of the received amplitudes from the cell of interest and A_I is the random variable of the received amplitudes from the interfering cells.

These formulas appear simplified in absence of fading and interfering cells in:

$$SIR_{AM} = \frac{1}{\frac{N_0}{2E_b} + \frac{1}{3G_p}\frac{E\left\{A_0^2\right\}}{\mu_A^2}(K_u - 1)} \tag{3.45}$$

$$BER_{AM} = Q\left(\sqrt{SIR_{AM}}\right). \tag{3.46}$$

3.2 Improved Gaussian Approximation (IGA)

The expressions in section 3.1 are only valid when the number of users is large. Furthermore, even when K_u is large, if the power control is not perfectly implemented, the MAI is not so accurately modelled as a Gaussian random variable. In situations where the Gaussian Approximation is not appropriate, a more in-depth analysis must be applied. In the previous section, the interference term I in (3.1) was a zero-mean Gaussian random variable with variance (2.15):

$$\sigma_I^2 = \frac{G_p T_c^2}{6} \sum_{k=1}^{K_u-1} P_k, \tag{3.47}$$

obtained by subsequently averaging the summation (2.7) over the distributions of the environmental parameters $\phi_k,\ \tau_k,\ B$. Thus, strictly speaking, the expressions of the SIR and BER for a single user channel are evaluated as a deterministic measure of averaged environmental conditions.

A different approach can be followed instead by calculating the averaged SIR and BER over the environmental statistics; it has been proved by simulations ([4], [5]) that this approach yields to an improvement of the Gaussian approximation method. This analysis defines the interference terms conditioned on the particular operating condition of each user. When this is done, ψ is defined as the conditioned variance of the multiple access interference for a specific operating condition [3]:

$$\psi = var\left(I|\left\{\varphi_k\right\},\left\{\tau_k\right\},\left\{P_k\right\},B\right) \tag{3.48}$$

hence the conditional variance of the MAI is itself a random variable; in this way, the SIR becomes

$$SIR|_\psi = \frac{P_0 T_b^2}{2\psi}. \tag{3.49}$$

A similar expression was derived in (3.5), with the fundamental difference that (3.5) evaluated the SIR assuming the average value of the variance of the multiple access interference term. On the other hand, if the distribution $f(\psi)$ of ψ is known, the SIR and the BER can be found by averaging overall possible values of ψ, as it can be seen in (3.50) and (3.51):

$$SIR_{IGA} = \int_0^\infty d\psi\ \frac{P_0 T_b^2}{2\psi}\ f(\psi) \tag{3.50}$$

$$BER_{IGA} = \int_0^\infty d\psi\ Q\left(\sqrt{\frac{P_0 T_b^2}{2\psi}}\right)\ f(\psi) \tag{3.51}$$

It has been demonstrated that, if the distribution of the interfering power levels is known, an analytical expression for $f(\psi)$ can be found [3], [15]. Moreover, in case of perfect power control, the technique shown in (3.50) and (3.51) yields to accurate results for a very small number of interfering users [9]. Note that this approach could be theoretically extended to all the equations presented in section 3.1, provided that an expression for $f(\psi)$ is available in those cases.

3.3 Simplified Expression of the Improvement Gaussian Approximation (SEIGA)

The expressions cited in the section 3.2 are complicated and required significant computational time to evaluate. Holtzman [6] presents a simplified technique for evaluating (3.50) and (3.51), that has been extended by Liberti [14] in the case of imperfect power control, and by Sunay and McLane [4] in the case of frequency non-selective fading channel.

The simplified SIR and BER expressions are based on the fact that if $g(x)$ is a continuous function and x is a random variable with mean value μ_x and variance σ_x^2, then the average value $E\{g(x)\}$ can be expressed by making use of the Taylor's expansion as [11]:

$$E\{g(x)\} = g(\mu_x + x) + \frac{1}{2}\sigma_x^2 g"(\mu_x) + \dots \tag{3.52}$$

Further computational savings can be obtained by expanding in differences rather than derivatives, so that

$$E\{g(x)\} \cong g(\mu_x) + \frac{\sigma_x^2}{2}\frac{g(\mu_x + h) - 2g(\mu_x) + g(\mu_x - h)}{h^2} \tag{3.53}$$

Choosing $h = \sqrt{3}\sigma_x$, (3.53) becomes [6]

$$E\{g(x)\} \cong \frac{2}{3}g(\mu_x) + \frac{1}{6}g\left(\mu_x + \sqrt{3}\sigma_x\right) + \frac{1}{6}g\left(\mu_x - \sqrt{3}\sigma_x\right). \tag{3.54}$$

Using this approximation, in the case of interference-limited case, the expression of SIR and BER are:

$$SIR_{SEIGA} \cong \frac{2}{3}\frac{P_0T_b^2}{2\mu_\psi} + \frac{1}{6}\frac{P_0T_b^2}{2\left(\mu_\psi + \sqrt{3}\sigma_\psi\right)} + \frac{1}{6}\frac{P_0T_b^2}{2\left(\mu_\psi - \sqrt{3}\sigma_\psi\right)} \tag{3.55}$$

$$BER_{SEIGA} \cong \frac{2}{3}Q\left(\sqrt{\frac{P_0T_b^2}{2\mu_\psi}}\right) + \frac{1}{6}Q\left(\sqrt{\frac{P_0T_b^2}{2\left(\mu_\psi + \sqrt{3}\sigma_\psi\right)}}\right) + \frac{1}{6}Q\left(\sqrt{\frac{P_0T_b^2}{2\left(\mu_\psi - \sqrt{3}\sigma_\psi\right)}}\right) \tag{3.56}$$

where μ_ψ is the mean value of the variance of the multiple access interference ψ, conditioned on a specific set of operating conditions, and σ_ψ^2 is the variance of ψ.

In absence of fading, if the received power levels from the $K_u - 1$ interfering users are identically distributed with mean μ_p and variance σ_p^2, the mean and variance of ψ are given by [3]:

$$\mu_\psi = \frac{T_c^2 G_p}{6}(K_u - 1)\mu_p \tag{3.57}$$

$$\sigma_\psi^2 = (K_u - 1)\frac{T_c^4}{4} \cdot \left[\frac{7G_p^2 + 2G_p - 2}{40}\sigma_p^2 + \left(\frac{23G_p^2}{360} + G_p\left(\frac{1}{20} + \frac{K_u - 2}{36}\right) - \frac{1}{20} - \frac{K_u - 2}{36}\right)\mu_p^2\right] \tag{3.58}$$

Note that equation (3.56) is valid only for

$$\mu_\psi > \sqrt{3}\sigma_\psi \tag{3.59}$$

to ensure that the denominator of the third term is positive. This leads to the following requirement

$$\frac{\sigma_p^2}{\mu_p^2} < \frac{10K_u(4G_p^2 - 3G_p + 3) - (109G_p^2 - 6G_p + 6)}{27(7G_p^2 + 2G_p - 2)}. \tag{3.60}$$

In the case where noise term is significant, the SIR and BER are given by

$$\begin{aligned} SIR_{SEIGA} &\cong \frac{2}{3}\frac{P_0T_b^2}{2\left(\mu_\psi + \frac{N_0T_b}{4}\right)} + \frac{1}{6}\frac{P_0T_b^2}{2\left(\mu_\psi + \sqrt{3}\sigma_\psi + \frac{N_0T_b}{4}\right)} \\ &\quad + \frac{1}{6}\frac{P_0T_b^2}{2\left(\mu_\psi - \sqrt{3}\sigma_\psi + \frac{N_0T_b}{4}\right)} \end{aligned} \tag{3.61}$$

$$\begin{aligned} BER_{SEIGA} &\cong \frac{2}{3}Q\left(\sqrt{\frac{P_0T_b^2}{2\left(\mu_\psi + \frac{N_0T_b}{4}\right)}}\right) + \frac{1}{6}Q\left(\sqrt{\frac{P_0T_b^2}{2\left(\mu_\psi + \sqrt{3}\sigma_\psi + \frac{N_0T_b}{4}\right)}}\right) + \\ &\quad \frac{1}{6}Q\left(\sqrt{\frac{P_0T_b^2}{2\left(\mu_\psi - \sqrt{3}\sigma_\psi + \frac{N_0T_b}{4}\right)}}\right) \end{aligned} \tag{3.62}$$

In this case the condition required for σ_p^2, given μ_p, is

$$\sigma_p^2 < \frac{\left(10K_u(4G_p^2 - 3G_p + 3) - (109G_p^2 - 6G_p + 6)\right)\mu_p^2 + \frac{120\mu_p G_p^2 N_0}{T_c^2} + \frac{90N_0^2G_p^2}{(K_u-1)T_c^4}}{27(7G_p^2 + 2G_p - 2)}. \tag{3.63}$$

When a frequency non-selective fading channel is considered, the expression of SIR and BER are given by [4]

$$\begin{aligned} SIR_{SEIGA}|_{\{\alpha_k\}} &\cong \frac{2}{3}\frac{P_0T_b^2\alpha_0^2}{2\left(\mu_\psi + \frac{N_0T_b}{4}\right)} + \frac{1}{6}\frac{P_0T_b^2\alpha_0^2}{2\left(\mu_\psi + \sqrt{3}\sigma_\psi + \frac{N_0T_b}{4}\right)} \\ &\quad + \frac{1}{6}\frac{P_0T_b^2\alpha_0^2}{2\left(\mu_\psi - \sqrt{3}\sigma_\psi + \frac{N_0T_b}{4}\right)} \end{aligned} \tag{3.64}$$

$$\begin{aligned} BER_{SEIGA}|_{\{\alpha_k\}} &\cong \frac{2}{3}Q\left(\sqrt{\frac{P_0T_b^2\alpha_0^2}{2\left(\mu_\psi + \frac{N_0T_b}{4}\right)}}\right) + \frac{1}{6}Q\left(\sqrt{\frac{P_0T_b^2\alpha_0^2}{2\left(\mu_\psi + \sqrt{3}\sigma_\psi + \frac{N_0T_b}{4}\right)}}\right) + \\ &\quad \frac{1}{6}Q\left(\sqrt{\frac{P_0T_b^2\alpha_0^2}{2\left(\mu_\psi - \sqrt{3}\sigma_\psi + \frac{N_0T_b}{4}\right)}}\right) \end{aligned} \tag{3.65}$$

where, in case of perfect power control, the mean and the variance of ψ are

$$\mu_{\psi}|_{\{\alpha_k\}} = \frac{G_p T_c^2 P_0}{6} \sum_{k=1}^{K_u-1} \alpha_k^2 \tag{3.66}$$

$$\sigma_{\psi}^2|_{\{\alpha_k\}} = \frac{T_c^4 P_0^2}{4} \left[\frac{23G_p^2 + 18G_p - 18}{360} \sum_{k=1}^{K_u-1} \alpha_k^4 + \frac{G_p - 1}{36} \sum_{k=1}^{K_u-1} \sum_{l=1\ l \neq k}^{Ku-1} \alpha_k^2 \alpha_l^2 \right] \tag{3.67}$$

In order to find the average SIR and BER for this case, expressions (3.64) and (3.65) have to be further averaged over the distribution of the path gains $\{\alpha_k\}$.

Note that equation (3.65) is only valid for

$$\frac{N_0 T_b}{4} + \frac{G_p}{6} T_c^2 (K_u - 1) P_0 > \sqrt{3}\sigma_{\psi} \tag{3.68}$$

to ensure that the term inside the square-root operator at the denominator of the third term is positive.

Chapter 4

Numerical results

In this chapter the results of a set of numerical tests obtained through the implementation of the equations of the previous chapter are presented. We consider different scenarios: the cases of a channel with or without fading and the cases of perfect or imperfect power control. The received power of the desired signal is normalized to 1. In all the simulations the chip rate R_c is equal to 3.84 Mbit/s, the Signal-to-Noise Ratio $E_b/N_0 = 10$ dB and the processing gain G_p is chosen among these values: 10, 64, 256. In the appendix A there are other numerical results with different values of R_c, G_p and E_b/N_0.

4.1 Channel without fading

4.1.1 Perfect power control

The first case we consider is a system for which the power control is perfect and the propagation channel is modeled without fading. In this case of SGA approximation we use the equations (3.11) and (3.12), for interference limited case (ilc), (3.9) and (3.10) for non-interference limited case (nilc). In this case of SEIGA approximation we use the equations (3.55) and (3.56), for interference limited case, (3.61) and (3.62) for non-interference limited case. To obtain perfect power control, we set μ_p=1 (3.57) and σ_p^2=0 (3.58).

In the following figures the SIR and the BER are plotted: in Fig. 4.1 $G_p = 10$, in Fig. 4.2 $G_p = 64$ and in Fig. 4.3 $G_p = 256$. We can observe, as expected, that the BER becomes significantly lower increasing G_p.

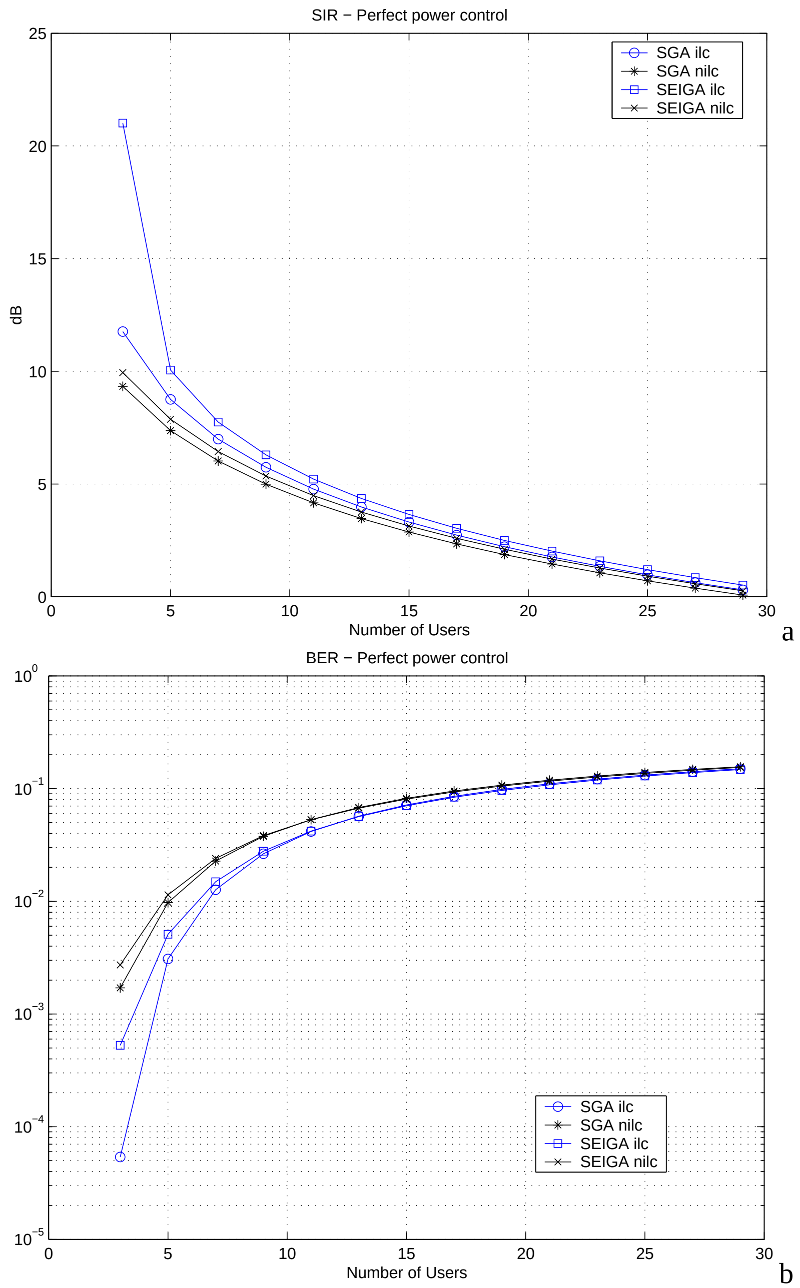

Figure 4.1: SIR (a) and BER (b) over a non-fading channel with perfect power control; $G_p = 10$.

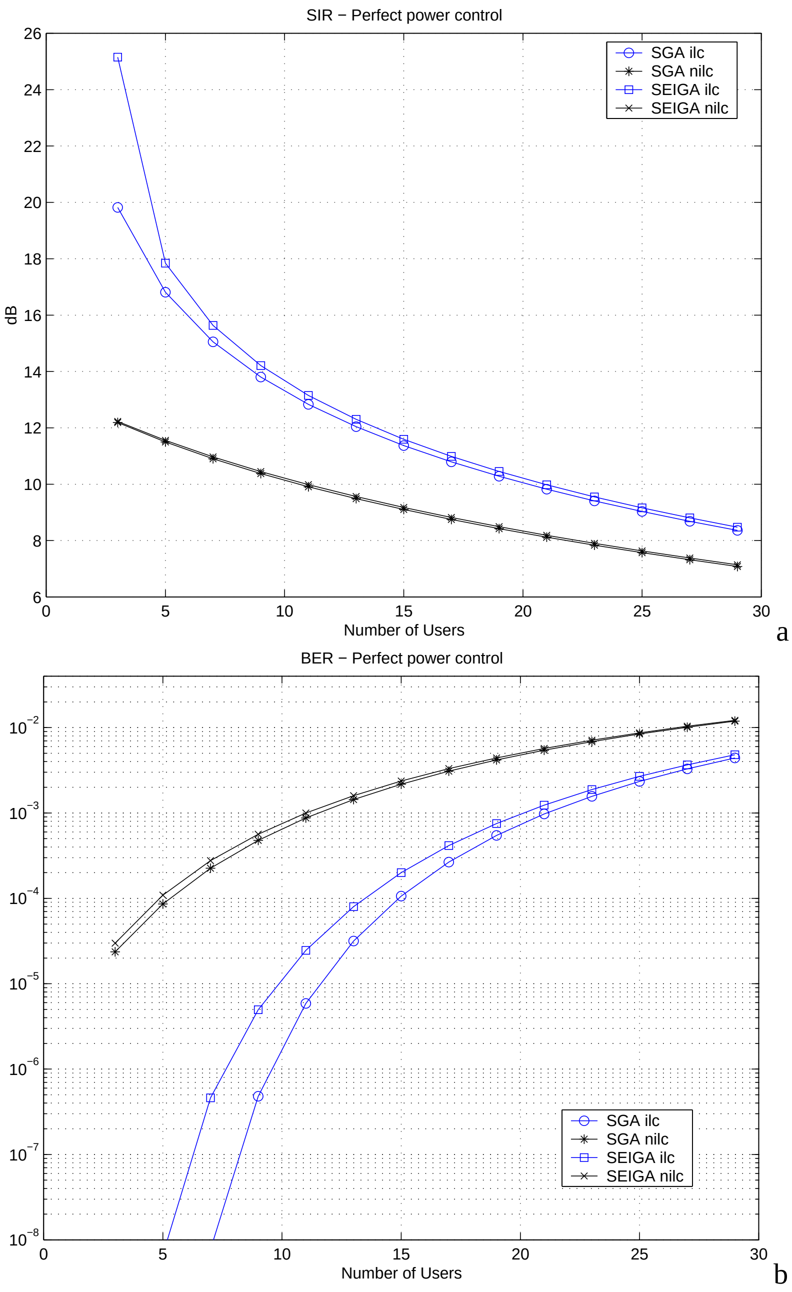

Figure 4.2: SIR (a) and BER (b) over a non-fading channel with perfect power control; $G_p = 64$.

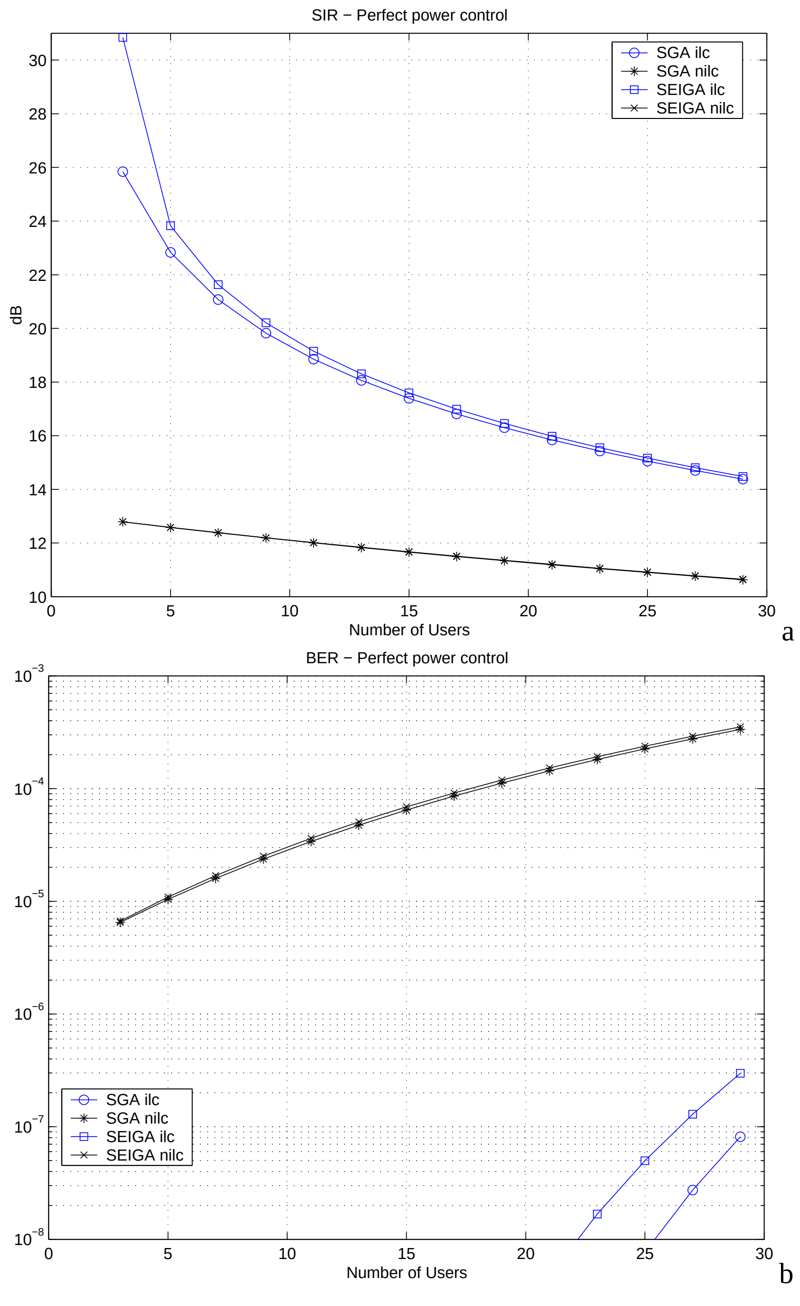

Figure 4.3: SIR (a) and BER (b) over a non-fading channel with perfect power control; $G_p = 256$.

4.1.2 Imperfect power control

In this section we analyze the effect of imperfect power control on the performance of a cellular CDMA, over a channel modeled without fading . When the power control is imperfect, the transmitted amplitude A_k of the kth user is a random variable. We consider a uniform distribution of the amplitude (3.27), where A_0 is the mean value and V is the maximum variation range of the received signal. With this assumption, the distribution of the power $P = A^2/2$ is [16]

$$f_P(p) = \frac{1}{\sqrt{2p}2V} \qquad \frac{(A_0 - V)^2}{2} < p < \frac{(A_0 + V)^2}{2} \tag{4.1}$$

So we obtain

$$\mu_p = \frac{(A_0 + V)^3 - (A_0 - V)^3}{12V} \tag{4.2}$$

$$\sigma_p^2 = \frac{(A_0 + V)^5 - (A_0 - V)^5}{40V} - \mu_p^2 \tag{4.3}$$

For the SGA approximation we use equations (3.32) and (3.33). For SEIGA approximation we use equations (3.61) and (3.62), putting in (3.57) and (3.58) the values of μ_p and σ_p^2 obtained in (4.2) and (4.3).

In the following figures the SIR and the BER are plotted: in Fig. 4.4 $G_p = 10$, in Fig. 4.5 $G_p = 64$ and in Fig. 4.6 $G_p = 256$. For reference we consider the same analysis with perfect power control (ppc, solid line with circle). Defining $k = V/A_0$, in solid line $k = 0.5$, in dash-dot line $k = 0.8$.

We can observe, as expected, that the BER is significantly lower increasing G_p. In all the analysis the performance of the system degrades respect to the case of perfect power control.

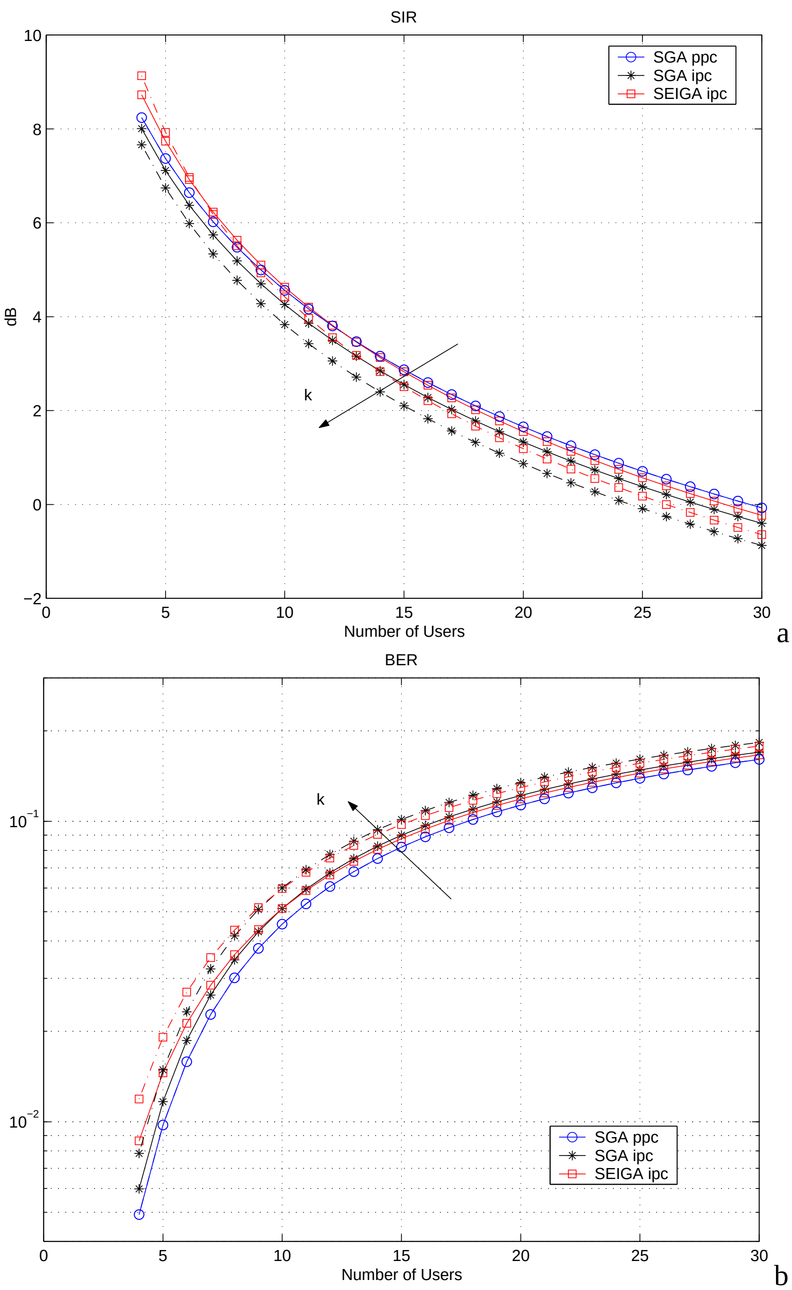

Figure 4.4: SIR (a) and BER (b) over a non-fading channel with imperfect power control (ipc); $G_p = 10$; k=0.5 in solid line; k=0.8 in dash-dot line

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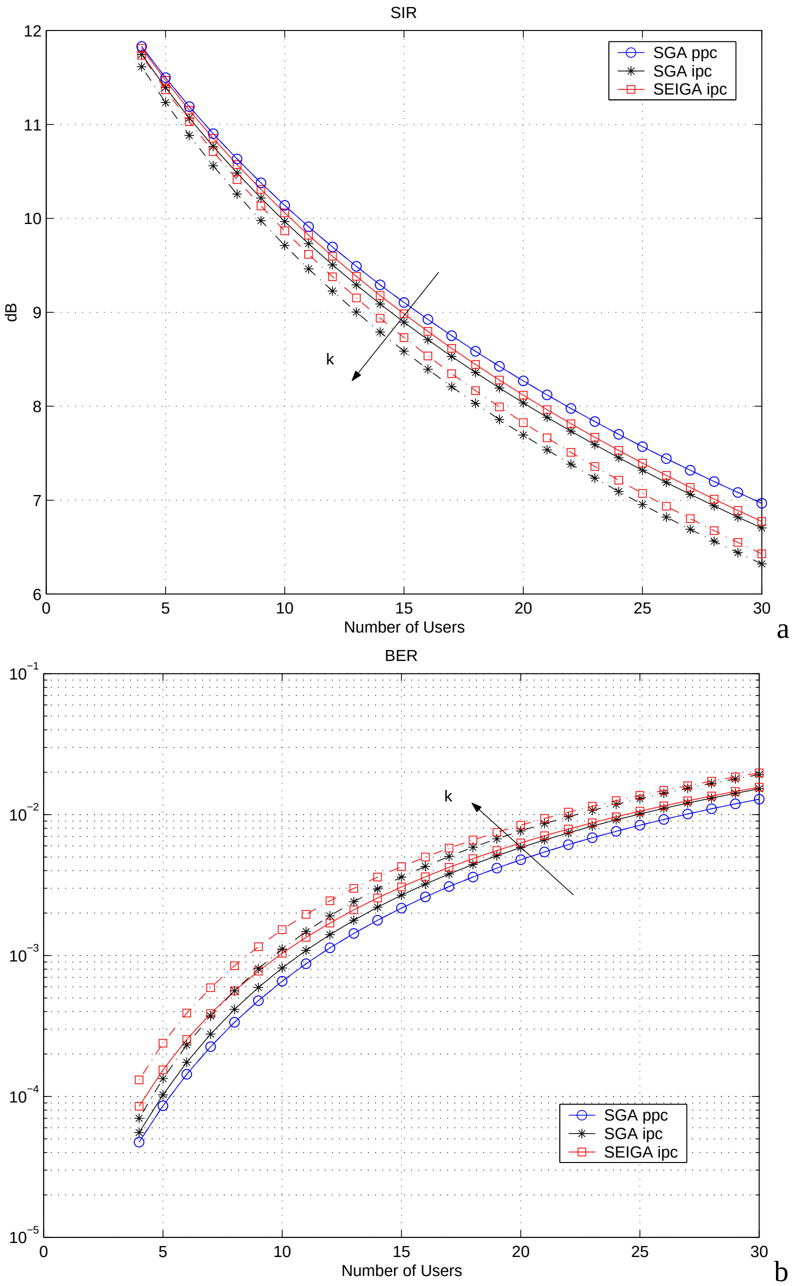

Figure 4.5: SIR (a) and BER (b) over a non-fading channel with imperfect power control (ipc); $G_p = 64$; k=0.5 in solid line; k=0.8 in dash-dot line

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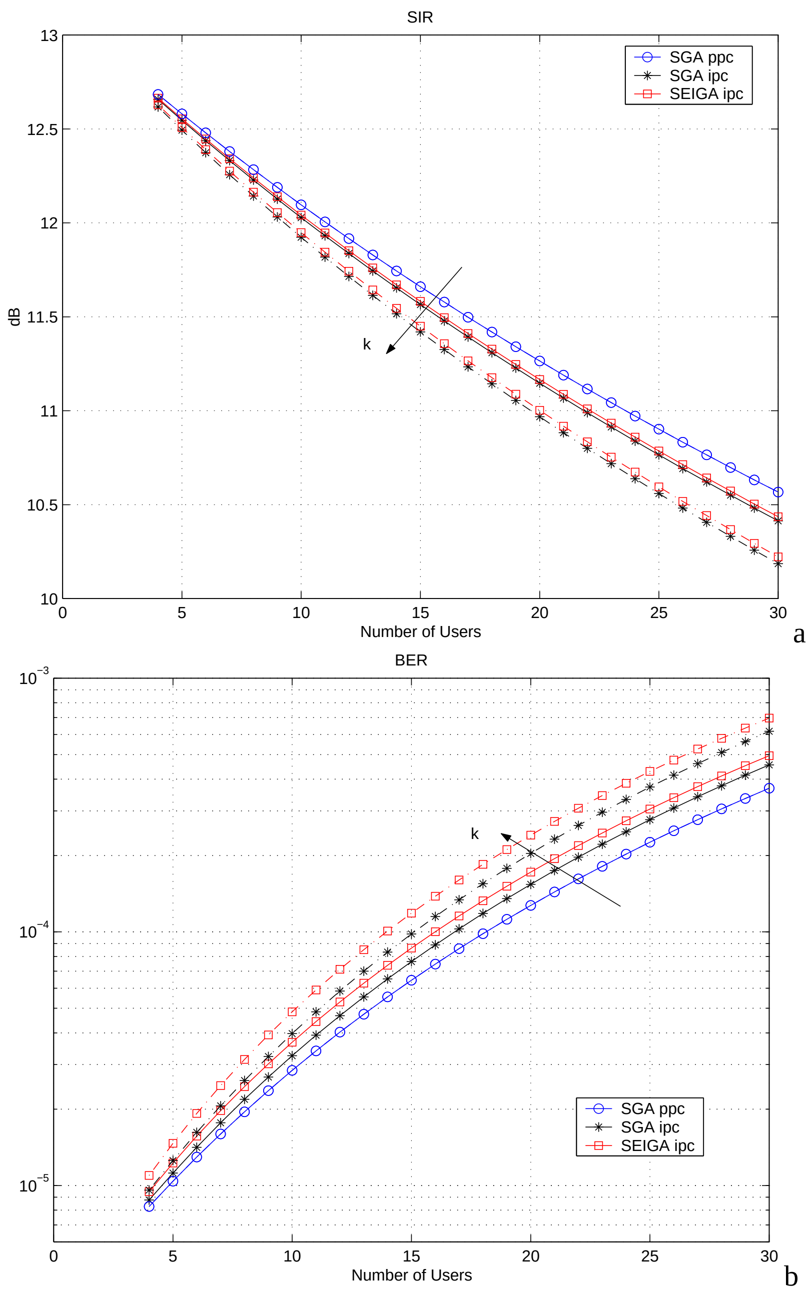

Figure 4.6: SIR (a) and BER (b) over a non-fading channel with imperfect power control (ipc); $G_p = 256$; k=0.5 in solid line; k=0.8 in dash-dot line

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4.1.3 Absence of power control

When the base station does not implement the power control, a *near-far* effect arises and the system performance significantly degrade, as it can be seen in figures 4.7, 4.8 and 4.9. The curves are obtained for different values of G_p by implementing formulas (3.9)-(3.10) for SGA method with perfect power control, formulas (3.32)-(3.33) for SGA method with imperfect power control, formulas (3.45)-(3.46) for SGA method with no power control, finally formulas (3.61)-(3.62) for SEIGA method in the cases of imperfect and absent power control, by means of (3.57)-(3.58) for μ_ψ and σ_ψ^2. In these last expressions, the values of μ_p and σ_p^2 are evaluated by means of (4.2)-(4.3) in case of imperfect power control, while in case of no power control, recalling that $P = A^2/2$, it is easy to obtain:

$$\mu_{p,apc} = E\left\{\frac{A^2}{2}\right\} = \frac{A_0^2}{2(R_c^2 - r_o^2)(n-1)}\left[r_o^{2-n} - R_c^{2-n}\right] \quad (4.4)$$

$$\sigma_{p,apc}^2 = E\left\{\left(\frac{A^2}{2}\right)^2\right\} - \mu_{p,apc}^2 = \frac{A_0^4}{4(R_c^2 - r_o^2)(n-1)}\left[r_o^{2(1-n)} - R_c^{2(1-n)}\right] - \mu_{p,apc}^2. \quad (4.5)$$

The figures have been obtained by setting the path loss exponent $n = 4$, the cell radius $R_c = 3$ km, and the minimum distance between transmitting and receiving antennas $r_o = 20$ m.

It must be noted that the SEIGA approximation for the BER cannot be calculated, because one of the terms under the squared root results always negative. However, one of the fundamental hypotheses under which the SEIGA approximation is developed, is that the system does perfectly implement the power control: this is evidently not our case. Therefore it is conceivable that in absence of power control not only the SEIGA approximation would lack in precision, but even its computation results impossible.

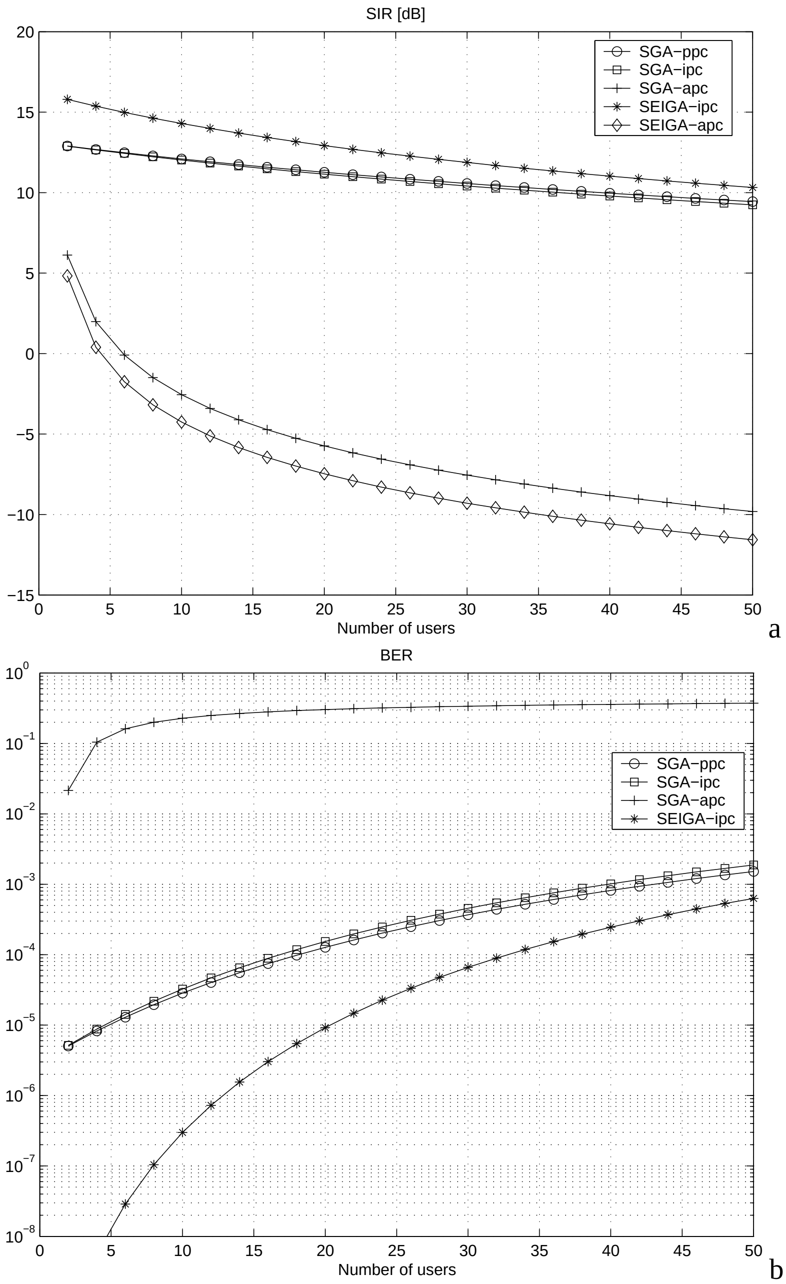


Figure 4.7: SIR (a) and BER (b) for $G_p = 256$, over a non-fading channel with perfect (ppc), imperfect (ipc) and absent power control (apc).

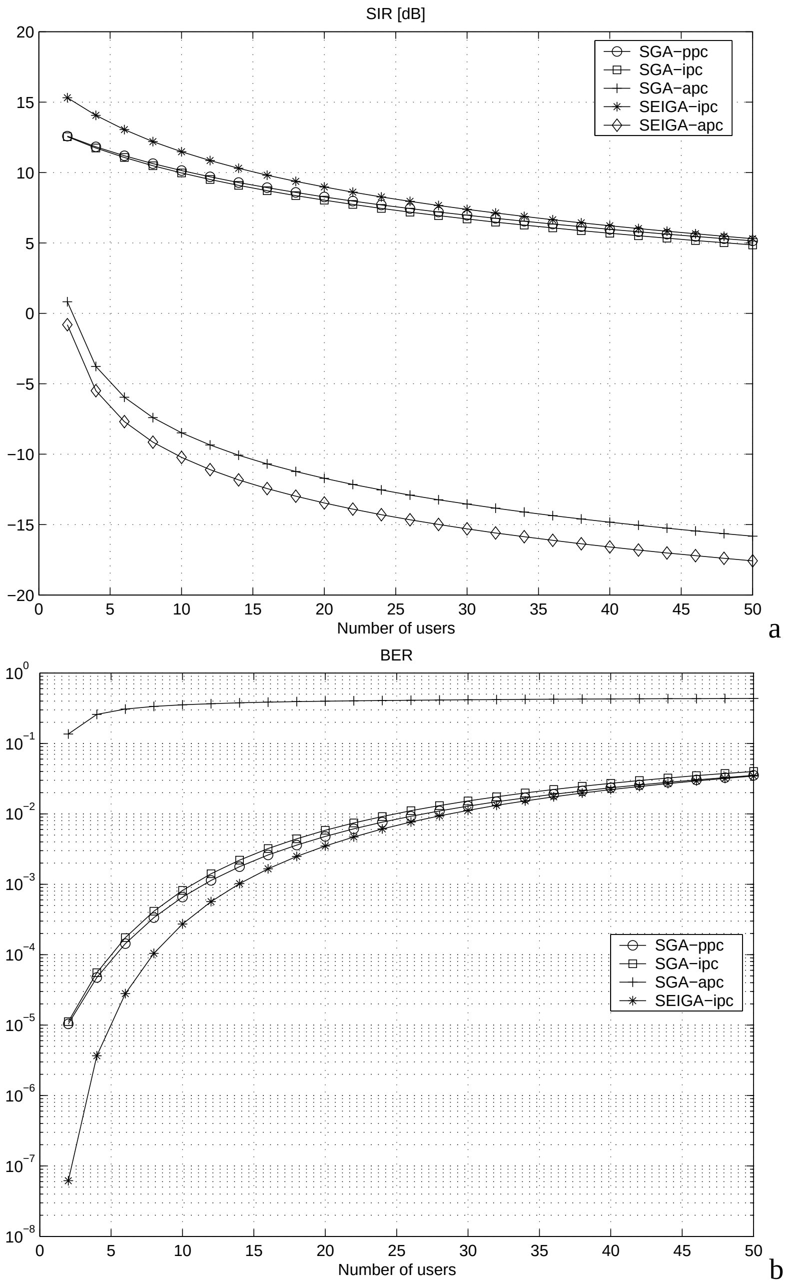


Figure 4.8: SIR (a) and BER (b) for $G_p = 64$, over a non-fading channel with perfect (ppc), imperfect (ipc) and absent power control (apc).

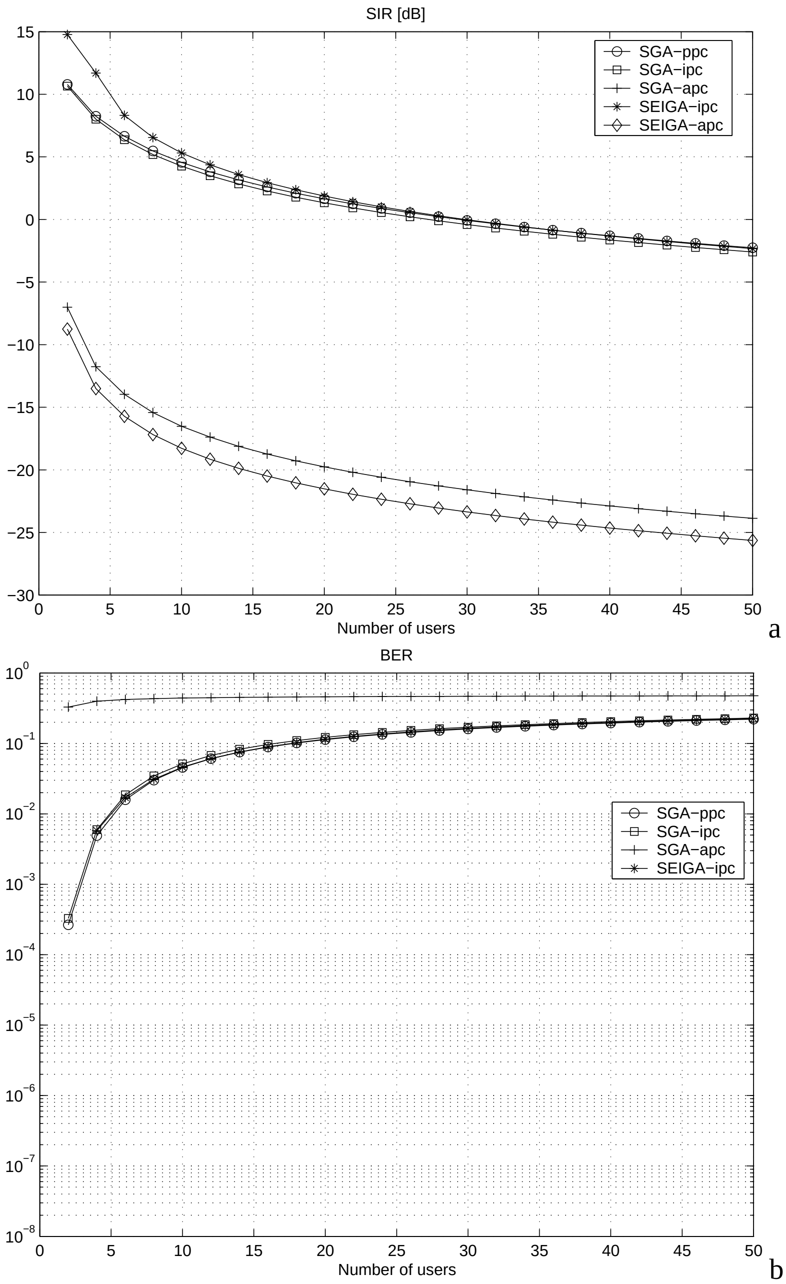


Figure 4.9: SIR (a) and BER (b) for $G_p = 10$, over a non-fading channel with perfect (ppc), imperfect (ipc) and absent power control (apc).

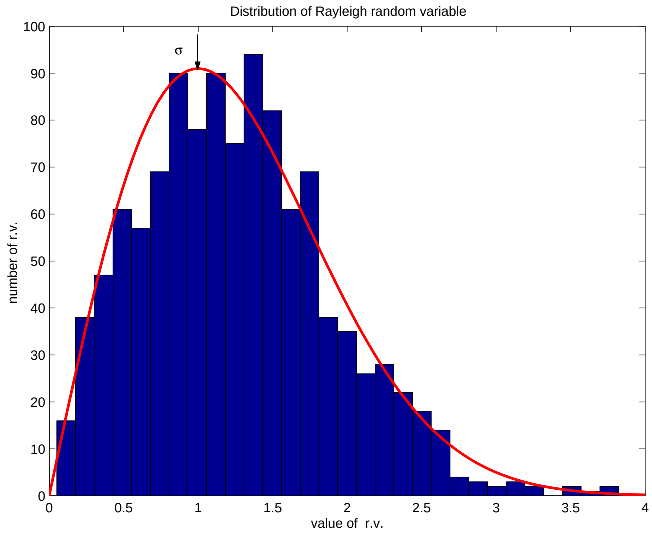


Figure 4.10: Distribution histogram of the variable used to model the fading. In solid line the analytical Rayleigh distribution.

4.2 Channel with fading

In this section we analyze the behavior of the system when the fading is modeled as a Rayleigh random variable with variance σ^2.

4.2.1 Perfect power control

First we consider the case of perfect power control. For SGA approximation we use two techniques through the equations (3.19) - (3.20) and (3.13) - (3.14) respectively, assuming $P_k = P_0$ for $k = 1, \ldots, K_u$.

In the first way, the fading process which independently affects all the users is described by means of its variance σ^2 only (3.19, 3.20).

In the second technique (3.13, 3.14), the independent fading processes are not averaged yet. To obtain the average SIR and average BER we use an intuitive montecarlo method: we repeat many realization with different set of variables with Rayleigh distribution and at last we average the result. The obtained result is an empirical estimate of the statistical approach. The figure 4.10 is an histogram of the values α_k of the used variable: the envelope is very similar to a Rayleigh distribution (solid line).

For SEIGA approximation, we use equations (3.64) and (3.65) by means of (3.66), (3.67). Also in this case the average value is obtained as a result of many simulations.

In Fig. 4.11, 4.12 and 4.13 the average SIR and average BER are plotted, respectively with $G_p = 10$, $G_p = 64$ and $G_p = 256$. In this analysis $\sigma^2 = 1$. The first SGA analysis is in solid line with circles, the second SGA analysis is in solid line with stars. To compare the two techniques, in (3.19) and (3.20) the number of interfering cells is equal to 0 and the number of multipaths is equal to 1: we can observe that the two results are very similar.

In all the cases we have a really higher BER respect to the case without fading.

Using equations (3.19) and (3.20), we analyze the performance of the system in presence of M multipaths and considering different number of interfering cells, through SGA approximation. In figure 4.14 the performance of a conventional correlation receiver are shown for a system with $G_p = 64$, $M = 4$ and $\sigma^2 = 1$.

Using equations (3.21) and (3.22), we analyze the case of a RAKE receiver. To obtain the results we numerically averaged the SIR and BER over the distribution of the random variable x (3.23). For the BER, the integration is made with the MATLAB-NAG routine d01amf. In figure 4.15 the performance of an M-finger RAKE receiver are shown, for a system with $G_p = 64$, $\sigma^2 = 1$, $M = 4$. Of course, we can observe that using a RAKE receiver the performance of the system are better than a conventional correlation receiver.

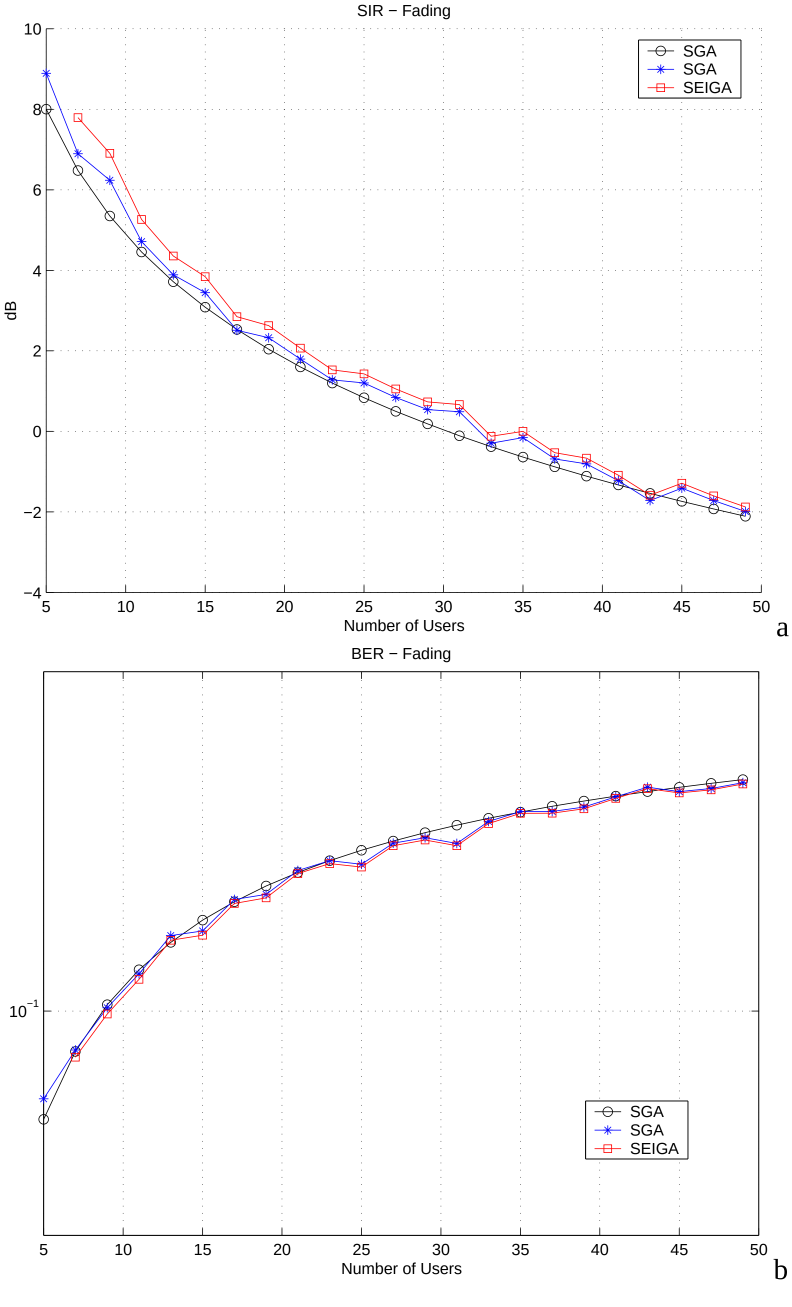


Figure 4.11: SIR (a) and BER (b) over a fading channel with perfect power control; $G_p = 10$.

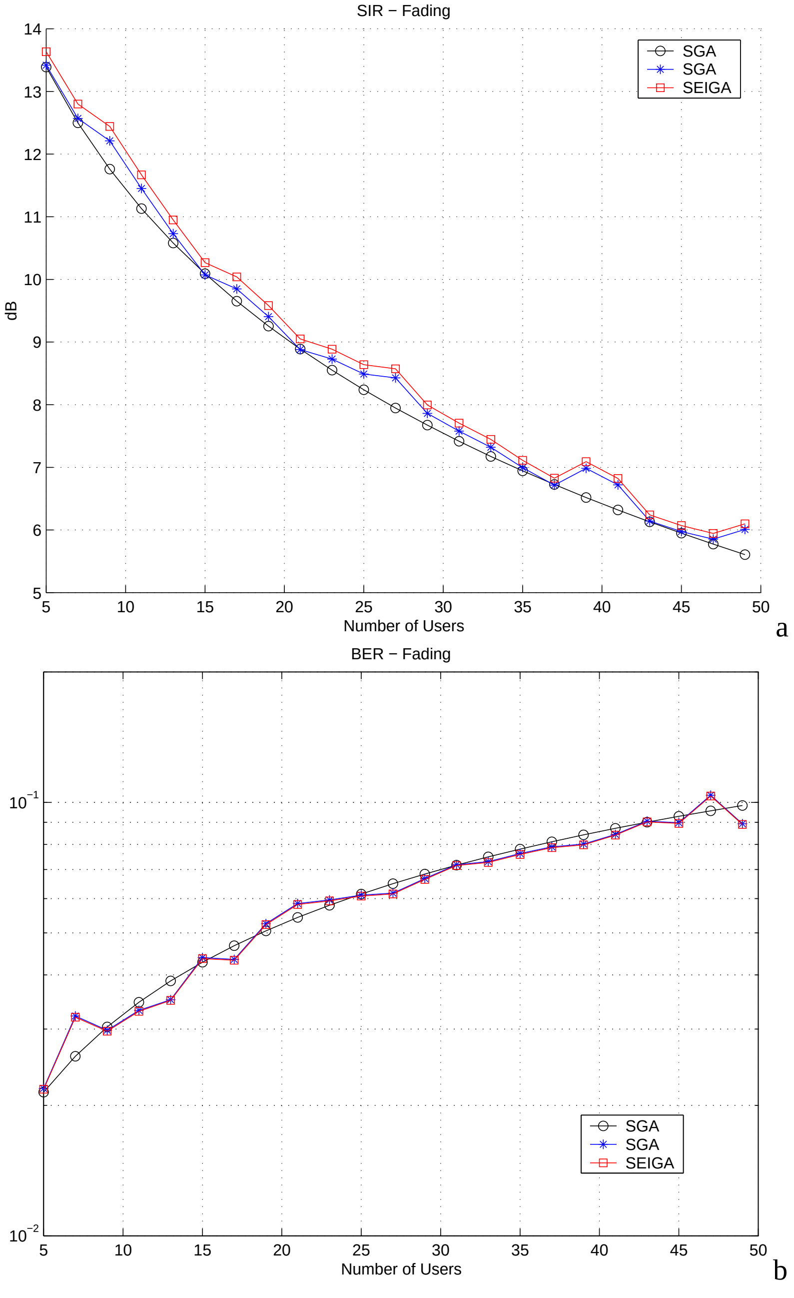


Figure 4.12: SIR (a) and BER (b) over a fading channel with perfect power control; $G_p = 64$.

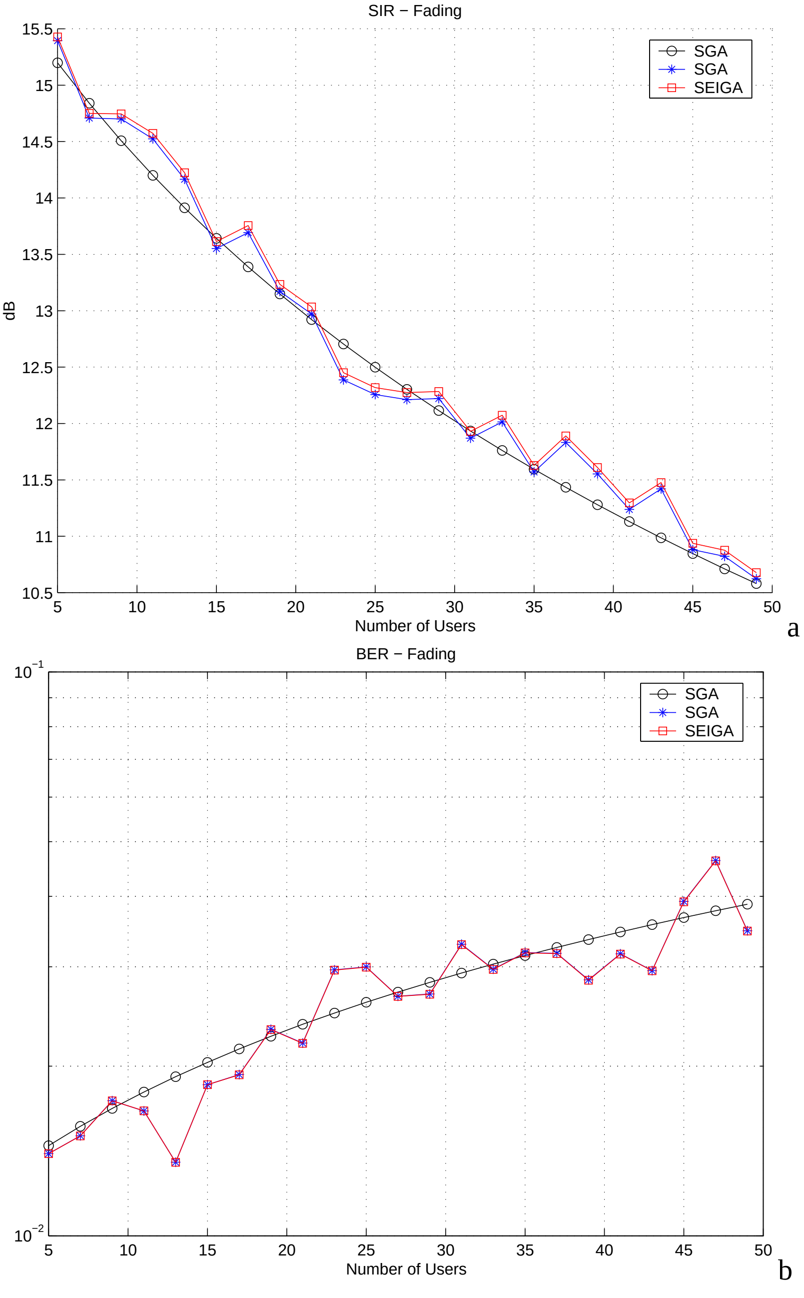


Figure 4.13: SIR (a) and BER (b) over a fading channel with perfect power control; $G_p = 256$.

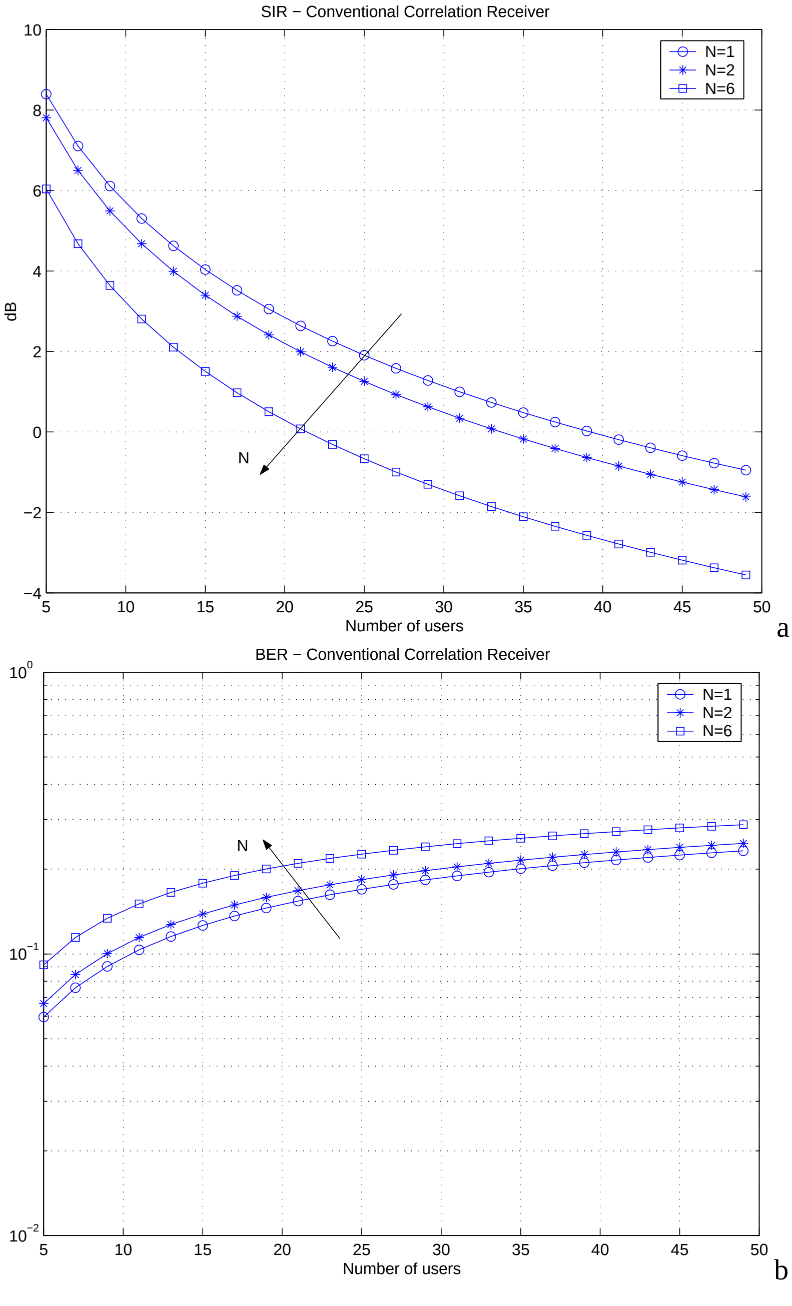


Figure 4.14: SIR (a) and BER (b) over a fading channel with perfect power control, as a function of the number of interfering cells N. Conventional correlation receiver, $G_p = 64$, number of multipath $M = 4$.

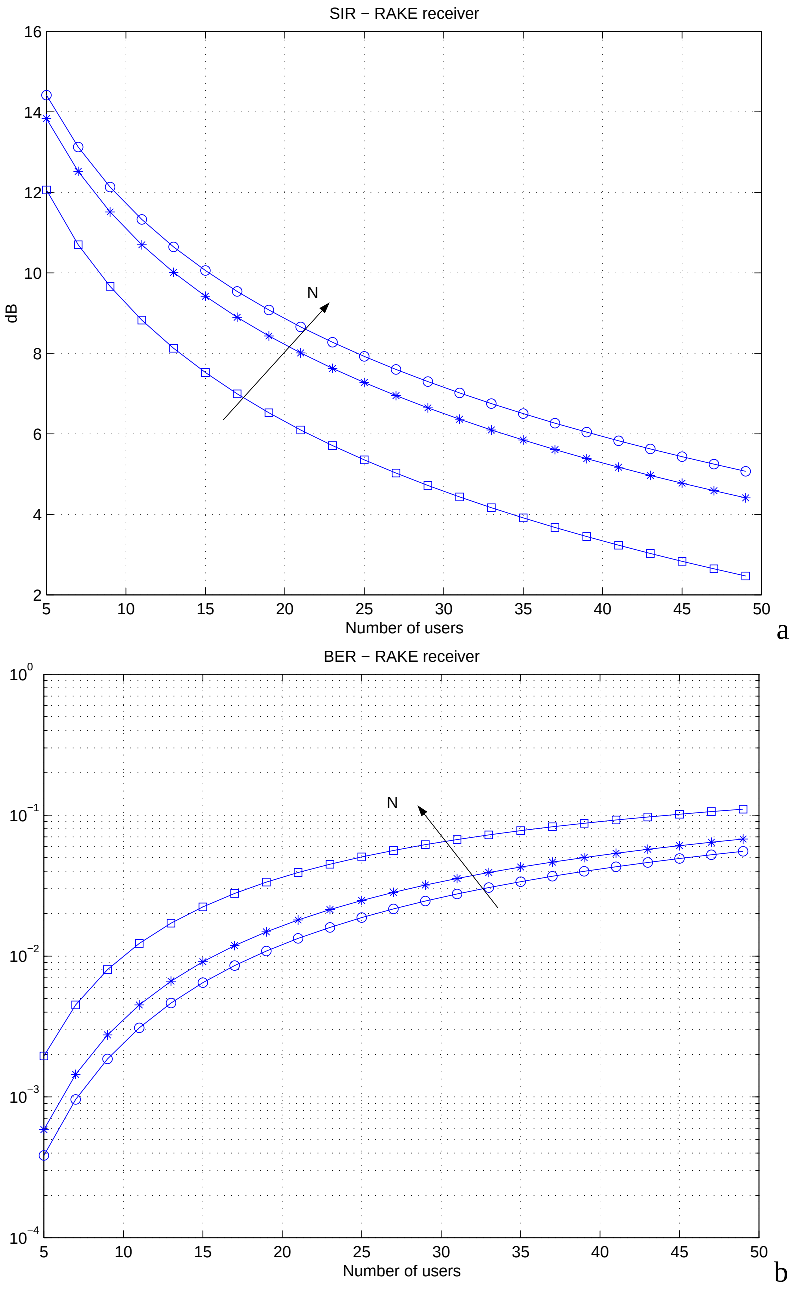


Figure 4.15: SIR (a) and BER (b) over a fading channel with perfect power control, as a function of the number of interfering cells N. RAKE receiver, $G_p = 64$, number of multipath $M = 4$.

4.2.2 Imperfect power control

In the analysis of a system over fading channel with imperfect power control, we consider the SGA approximation through the equations (3.28) and (3.29) for a conventional correlation receiver, equations (3.30) and (3.31) for a RAKE receiver. In both cases we consider the interference of multipaths and users of other cells. The imperfect power control is modeled as a uniform distribution of the amplitude of the received signal (3.27), where $k = V/A_0$. In figure 4.16 we consider a conventional correlation receiver and $G_p = 64$, $M = 4$, $\sigma^2 = 1$, as a function of the number of interfering cells. There are two cases of imperfect power control: $k = 0.5$ in solid line; $k = 0.8$ in dash-dot line. In figure 4.17 there is the same analysis with a RAKE receiver. The performance of a RAKE receiver are better than a conventional correlation receiver, and, in both cases, the behavior is as worse as more imperfect is the power control (i.e. as k increases).

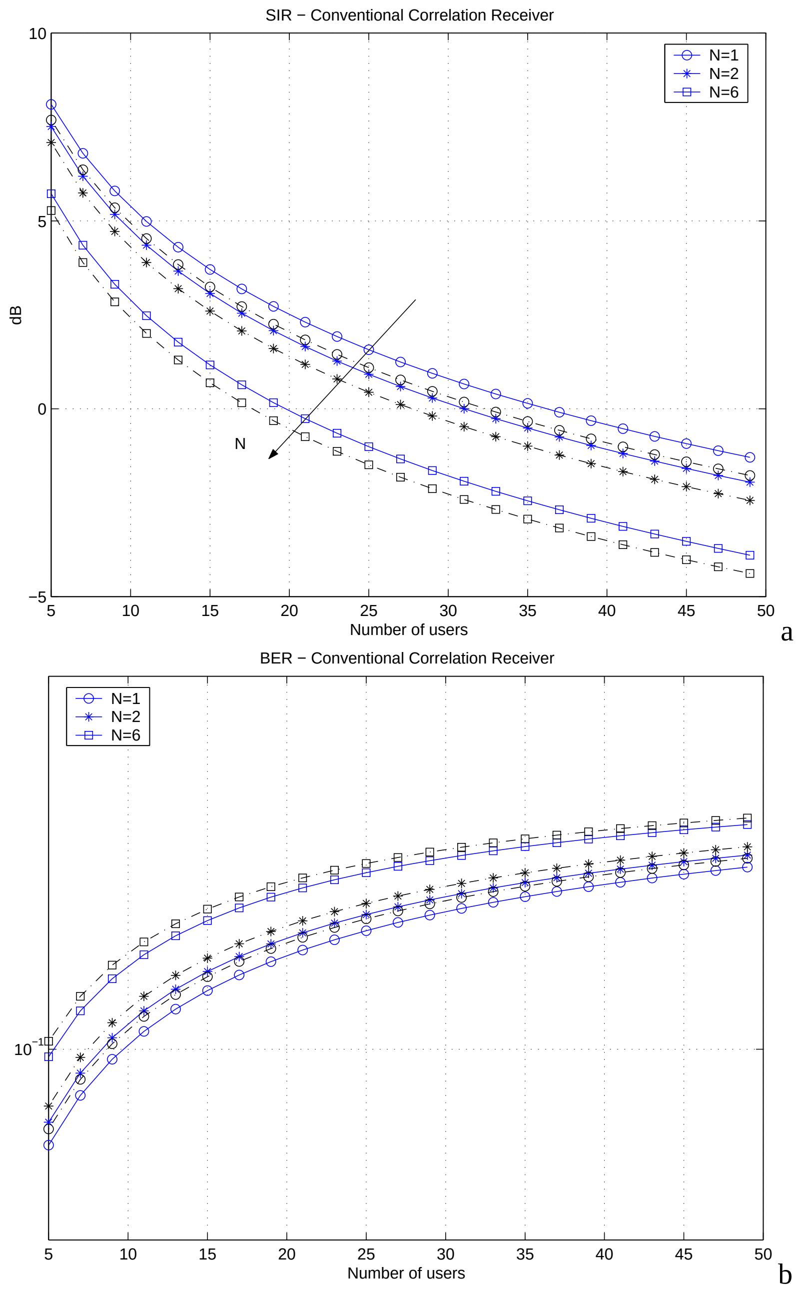


Figure 4.16: SIR (a) and BER (b) over a fading channel with imperfect power control, as a function of the number of interfering cells N; $\sigma^2 = 1$, $G_p = 64$, number of multipaths $M = 4$. Conventional correlation receiver; $k = 0.5$ in solid line; $k = 0.8$ in dash-dot line.

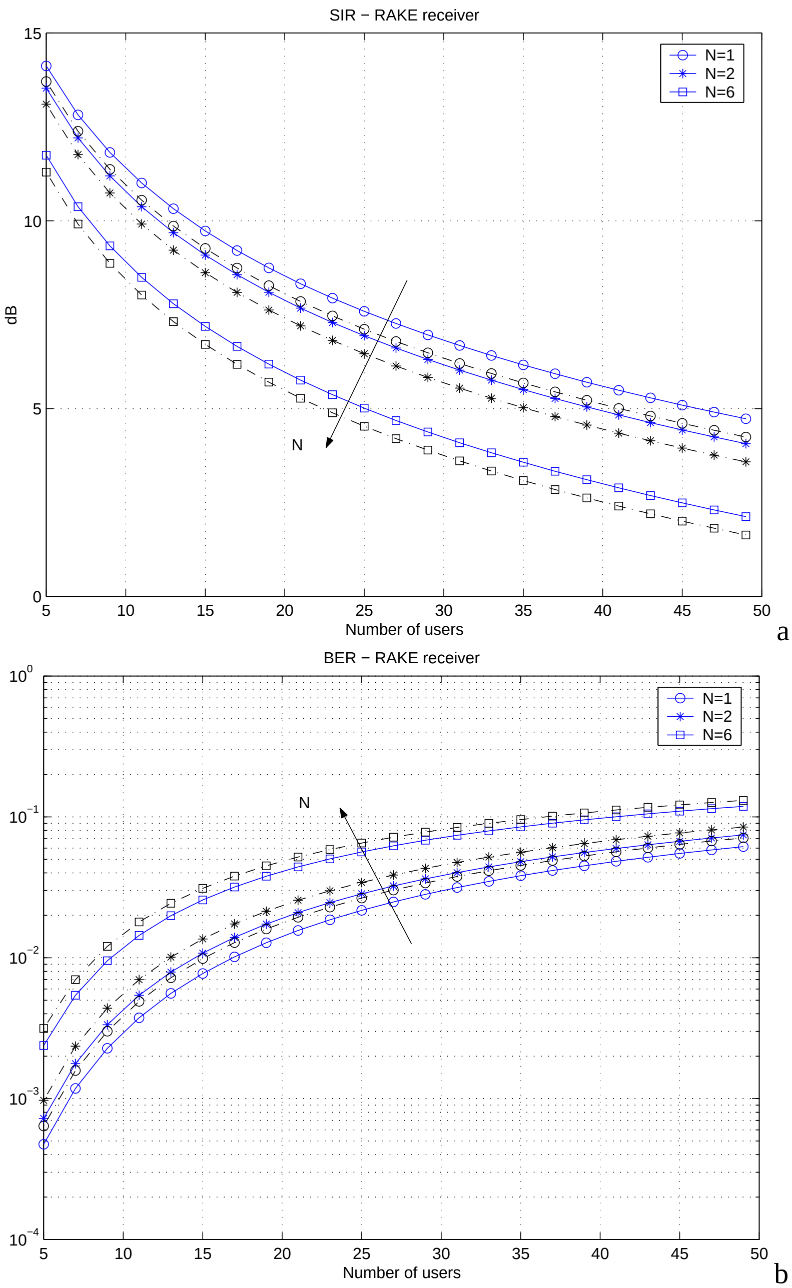


Figure 4.17: SIR (a) and BER (b) over a fading channel with imperfect power control, as a function of the number of interfering cells N; $\sigma^2 = 1$, $G_p = 64$, number of multipaths $M = 4$. RAKE receiver; $k = 0.5$ in solid line; $k = 0.8$ in dash-dot line.

4.2.3 Absence power control

A comparison of the system performance over a 4-multipath channel with perfect, imperfect and no power control and variable processing gain is shown in figures 4.18, 4.19 and 4.20, for conventional and RAKE receivers and SGA approximations. The implemented formulas for the case of no power control are (3.43) and (3.44). As expected, in absence of power control the performance severely degrade; however, in this case the Gaussian approximation could provide very imprecise estimates.

The figures have been obtained by setting: $\sigma^2 = 1$ for the Rayleigh fading, $M = 4$ multipath rays, $k = 0.5$ for the imperfect power control, path loss exponent $n = 4$ when no power control is implemented, cell radius $R_c = 3$ km, minimum distance between transmitting and receiving antennas $r_o = 20$ m, number of interfering cells $N_c = 6$.

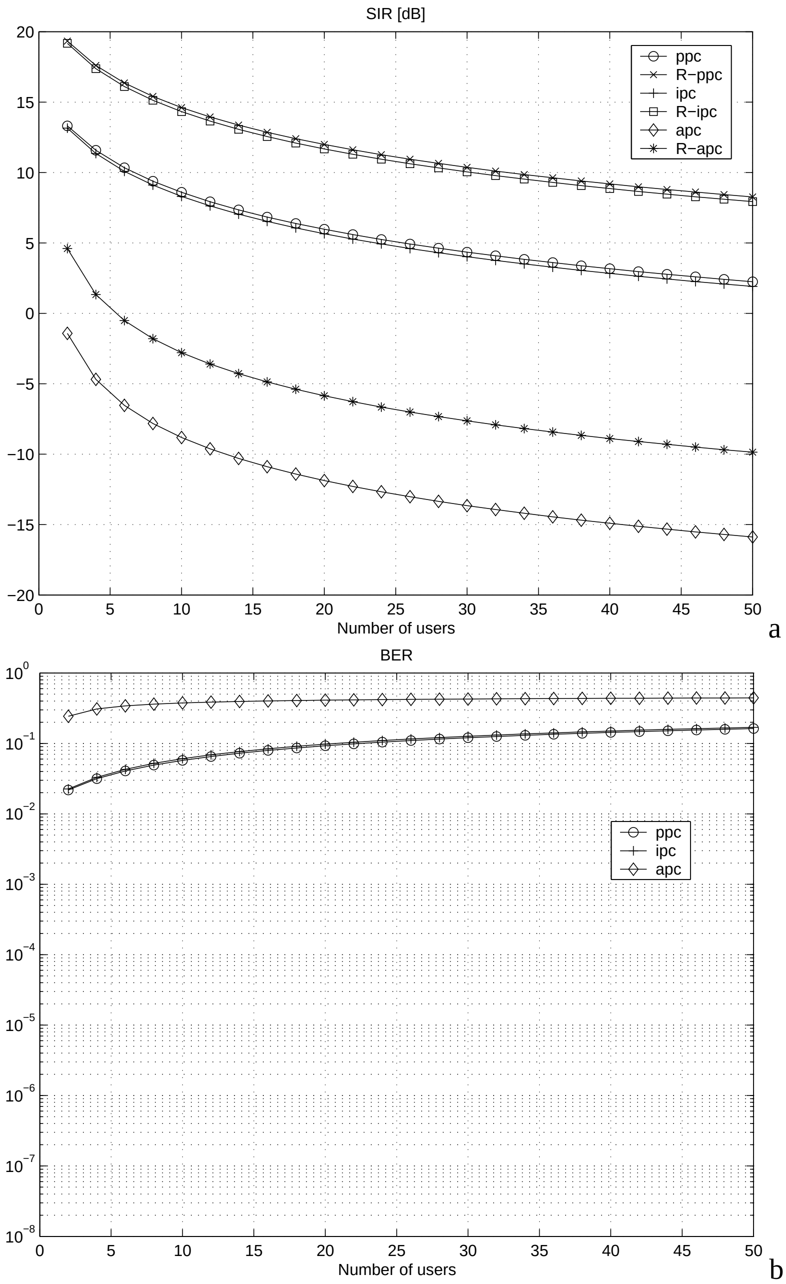


Figure 4.18: SIR (a) and BER (b) for $G_p = 256$, over a fading channel with perfect (ppc), imperfect (ipc) and no power control (apc), with conventional and RAKE receiver (R).

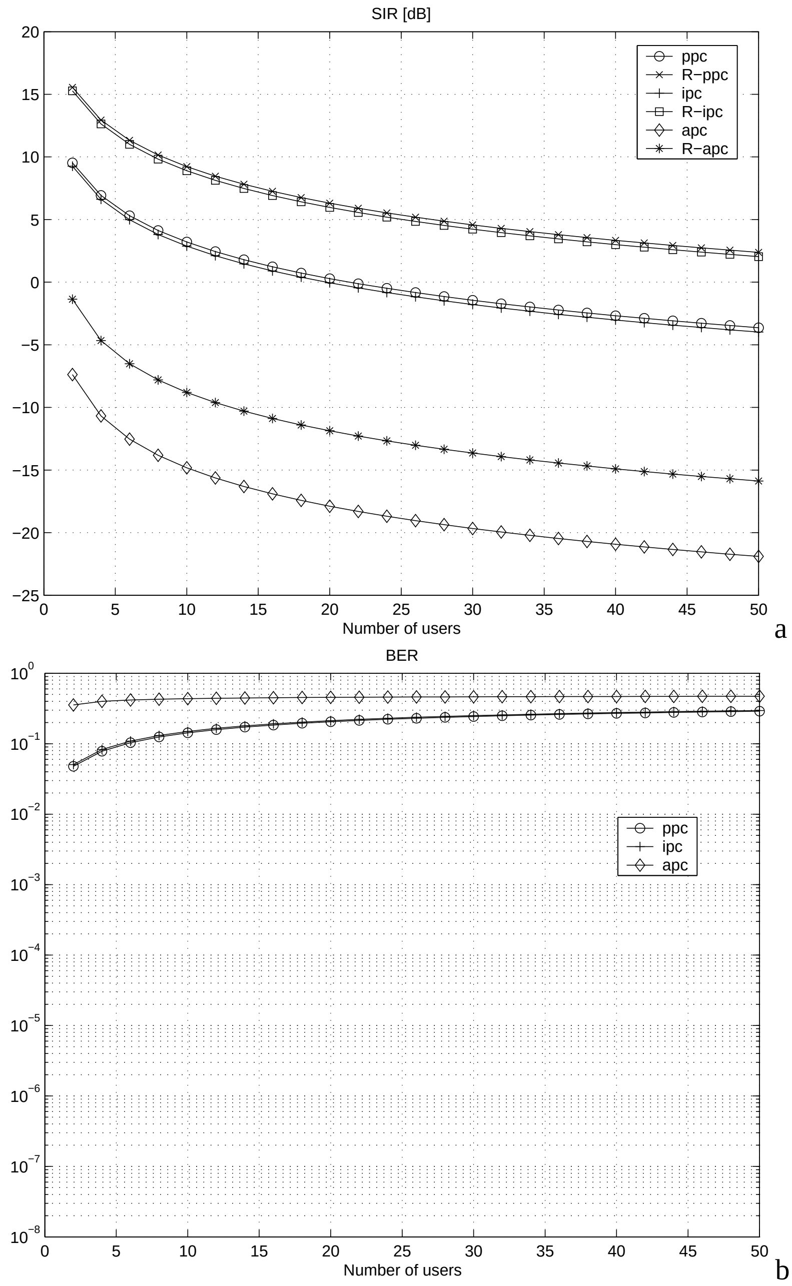
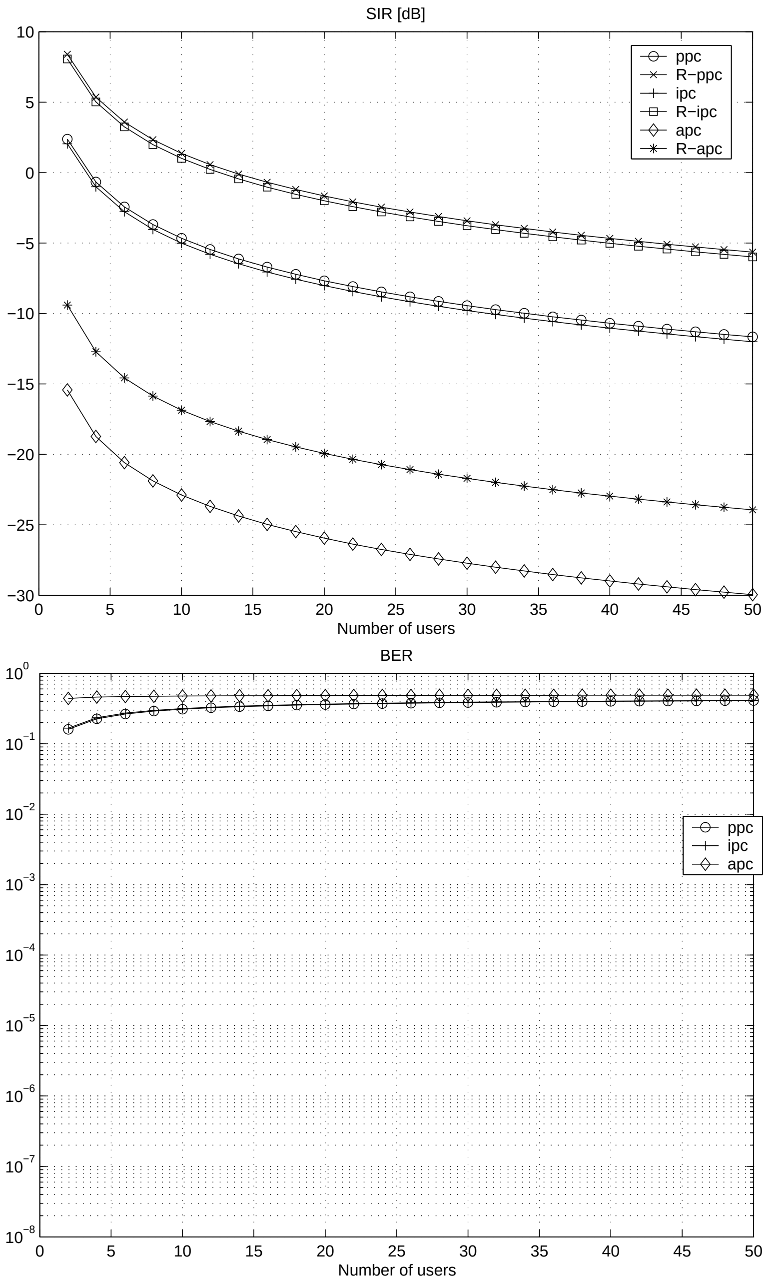


Figure 4.19: SIR (a) and BER (b) for $G_p = 64$, over a fading channel with perfect (ppc), imperfect (ipc) and no power control (apc), with conventional and RAKE receiver (R).



a

b

Figure 4.20: SIR (a) and BER (b) for $G_p = 10$, over a fading channel with perfect (ppc), imperfect (ipc) and no power control (apc), with conventional and RAKE receiver (R).

Chapter 5

Conclusions

The problem of describing the interference generated by a number of co-channel independent sources in a synchronous or asynchronous DS-CDMA system is made a complex task by the inherent unpredictability of the wireless communication scenario. The Gaussian approximation, by mean of the Central Limit Theorem theory, leads to a fundamental simplification of the problem formulation, thus allowing an analytical development and a very computationally efficient solution for the system performance estimate in terms of SIR and BER, based on the statistical description of all the channel impairments. In this way, tedious and cost-inefficient simulations can be avoided.

Several improvements have been proposed for the standard method (IGA, SEIGA), that slightly better behave in case of reduced number of interferers and/or imperfect power control. In fact, in literature SGA methods showed inaccurate and optimistic results with respect to simulations, in terms of BER; our results confirm that in all cases SEIGA methods achieve higher BER levels, in particular for a few interferers and imperfect power control. However, the application of SEIGA expressions is limited by some numerical constraints, which made the use of this method critical for some situations.

As far as the SIR is concerned, for the best of our knowledge literature does not present results comparing different Gaussian method applied to the SIR evaluation. Out developments experienced slightly higher SIR level estimates for all the SEIGA expressions.

It must be noted that both the SGA an SEIGA expressions we developed for the case of absent power control are borderline cases: since the power levels at the receiver can be very different, the Gaussian hypothesis from the Cental Limit Theorem can be very hardly accounted for in that case.

Although the Gaussian hypothesis could appear a strained operation for some situations (for *real situations*), it seems however the unique practical, very flexible and very computationally efficient method that has been provided in order to give an estimate of the interference effects in DS-CDMA systems.

Appendix A

Other numerical results

In this appendix there is a collection of other numerical results, similar to the simulations of chapter 4. In these analysis $R_b = 9600$ bit/s, $E_b/N_0 = 20$ dB and G_p is equal to 32 or 128. Many different scenarios are considered: channel with or without fading; perfect, imperfect or absence of power control.

Channel without fading and Perfect power control

In figure A.1 the SIR and the BER are plotted; $G_p = 32$.

In figure A.2 there is the same analysis with $G_p = 128$. We can observe, as expected, that the BER is significantly lower than in the previous case.

Channel without fading and Imperfect power control

In figure A.3 the SIR and the BER are plotted; in this analysis $G_p = 32$. For reference we consider the same analysis with perfect power control (ppc, solid line with circle). Defining $k = V/A_0$, in solid line $k = 0.1$, in dash-dot line $k = 0.5$.

In figure A.4 there is the same analysis with $G_p = 128$. We can observe, as expected, that the BER is significantly lower than the previous case.

In both analysis the performance of the system degrades respect to the case of perfect power control.

Channel without fading and Absence of power control

Figure A.5 has been obtained by setting the following system parameters: $G_p = 128$, path loss exponent $n = 4$, cell radius $R_c = 3$ km, minimum distance between transmitting and receiving antennas $r_o = 20$ m.

Channel with fading and Perfect power control

In figure A.6 the average SIR and average BER are plotted. In this analysis $G_p = 32$ and $\sigma^2 = 1$. The first SGA analysis is in solid line with circles, the second SGA analysis is in solid line with stars. To

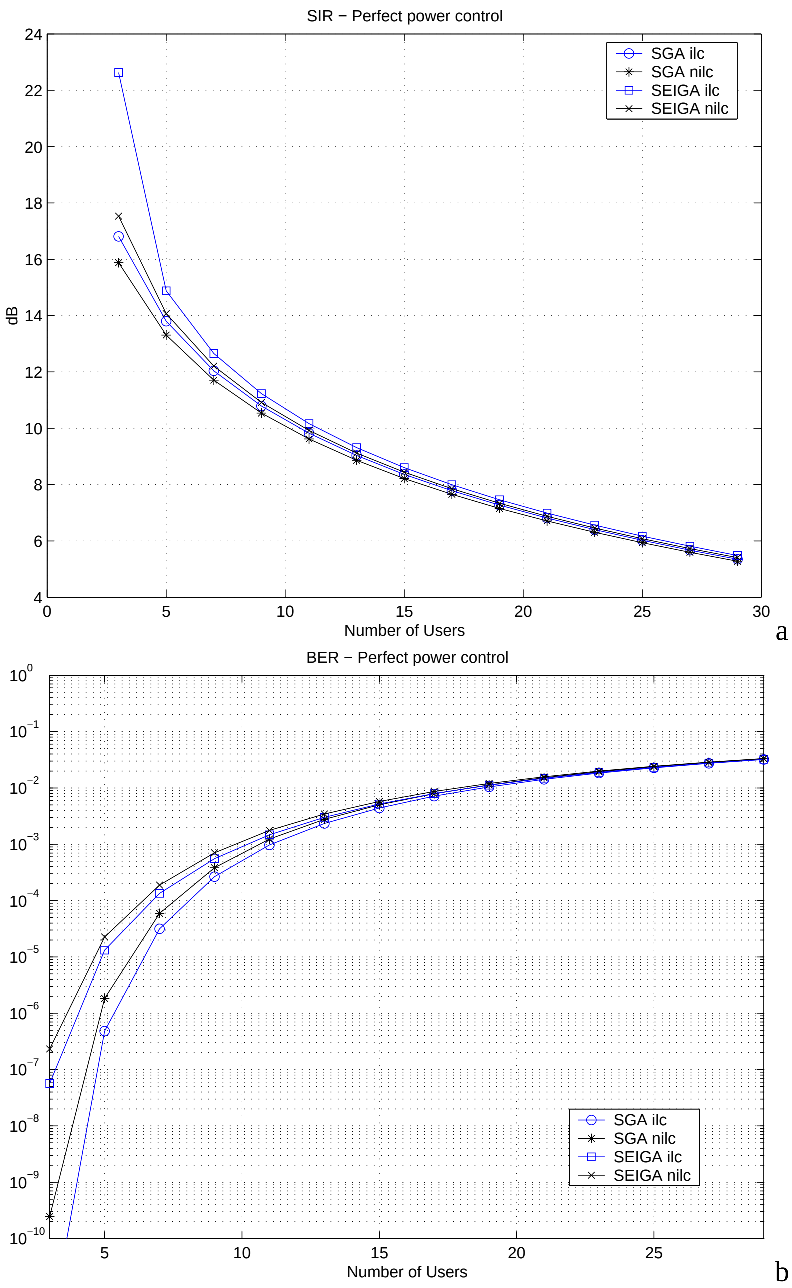
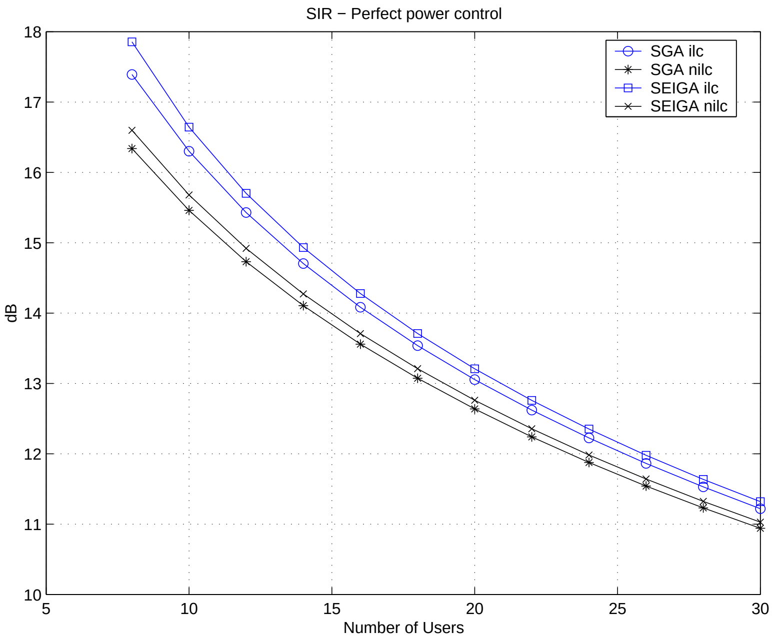


Figure A.1: SIR (a) and BER (b) over a non-fading channel with perfect power control; $G_p = 32$.

compare the two techniques, in (3.19) and (3.20) the number of interfering cells is equal to 0 and the number of multipaths is equal to 1: we can observe that the two results are very similar.



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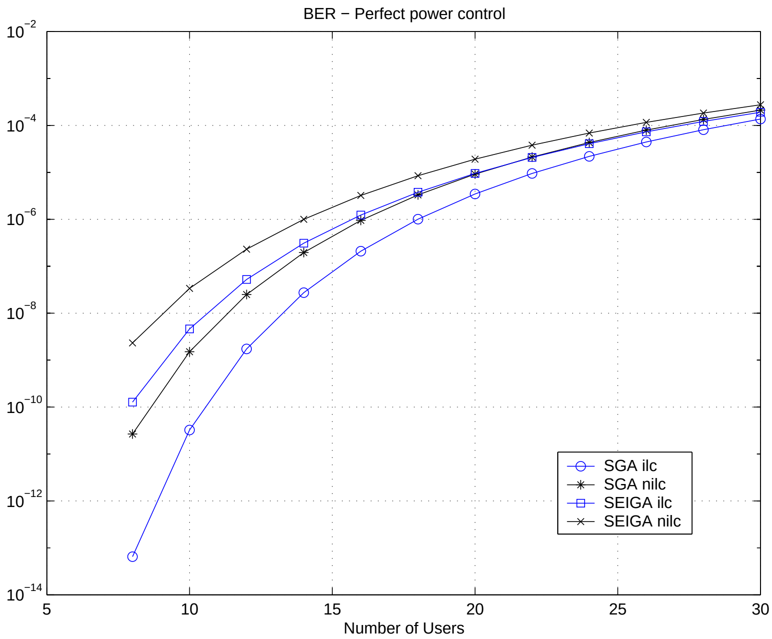

b

Figure A.2: SIR (a) and BER (b) over a non-fading channel with perfect power control; $G_p = 128$.

In figure A.7 there is the same analysis with $G_p = 128$.
In both cases we have a really higher BER respect to the case without fading.

In figure A.8 the performance of a conventional correlation receiver are shown for a system with $G_p = 128$, $M = 4$ and $\sigma^2 = 1$.

In figure A.9 the performance of an M-finger RAKE receiver are shown, for a system with $G_p = 128$, $\sigma^2 = 1$, $M = 4$. Of course, we can observe that using a RAKE receiver the performance of the system are better than a conventional correlation receiver.

Channel with fading and Imperfect power control

In figure A.10 we consider a conventional correlation receiver and $G_p = 128$, $M = 4$, $\sigma^2 = 1$, as a function of the number of interfering cells. There are two cases of imperfect power control: $k = 0.5$ in solid line; $k = 0.8$ in dash-dot line. In figure A.11 there is the same analysis with a RAKE receiver. The performance of a RAKE receiver are better than a conventional correlation receiver, and, in both cases, the behavior is as worse as more imperfect is the power control (i.e. as k increases).

Channel with fading and Absence power control

A comparison of the system performance over a 4-multipath channel with perfect, imperfect and no power control is shown in figure A.12, for conventional and RAKE receivers and SGA approximations. The figure has been obtained by setting the following system parameters: $G_p = 128$, $\sigma^2 = 1$, $M = 4$, $k = 0.5$, path loss exponent $n = 4$, cell radius $R_c = 3$ km, minimum distance between transmitting and receiving antennas $r_o = 20$ m, number of interfering cells $N_c = 6$.

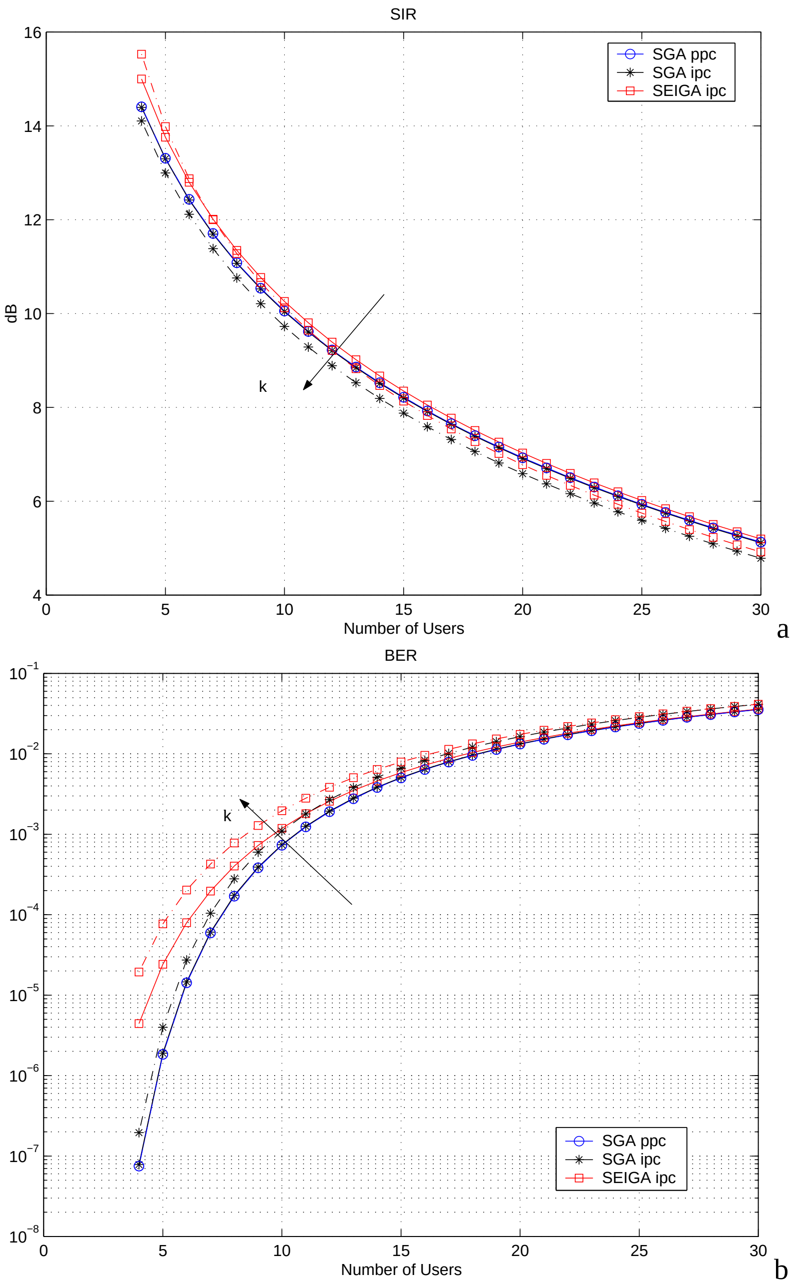

Figure A.3: SIR (a) and BER (b) over a non-fading channel with imperfect power control (ipc); $G_p = 32$; k=0.1 in solid line; k=0.5 in dash-dot line

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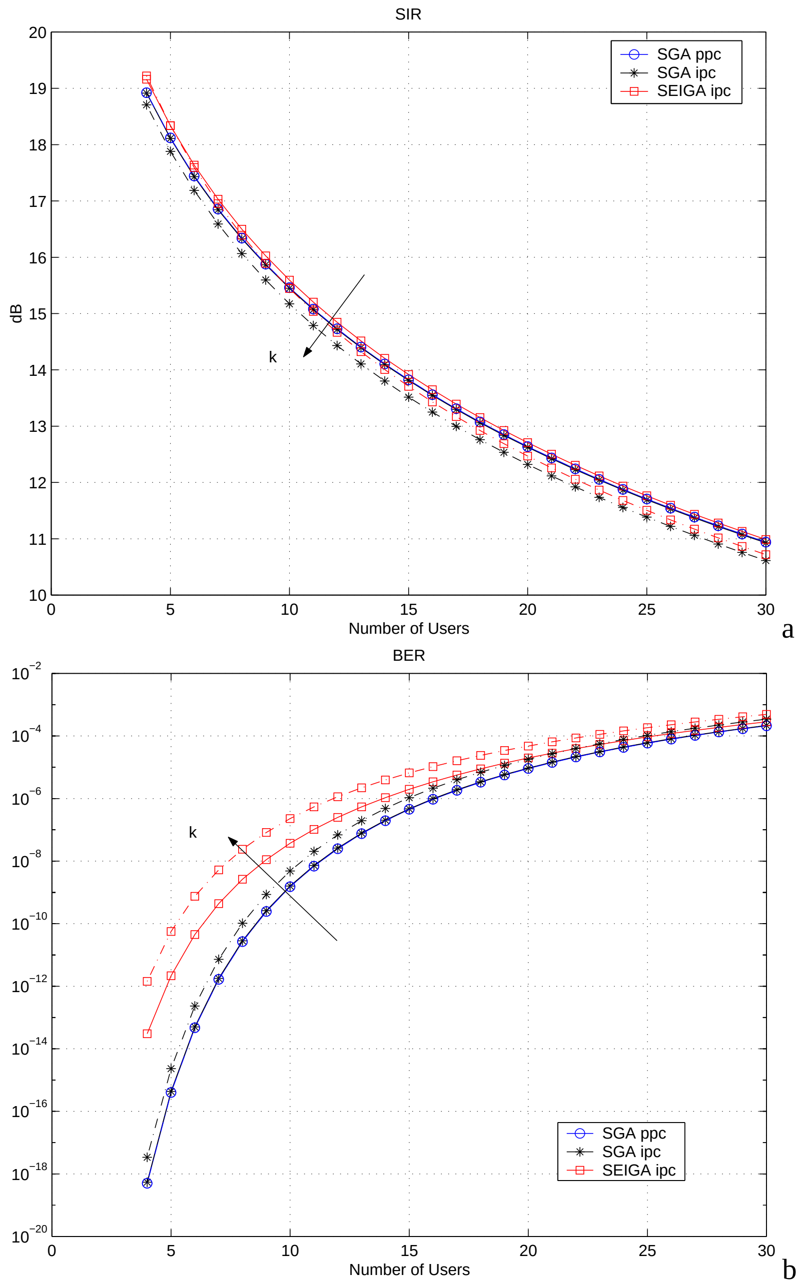

Figure A.4: SIR (a) and BER (b) over a non-fading channel with imperfect power control (ipc); $G_p = 128$; k=0.1 in solid line; k=0.5 in dash-dot line

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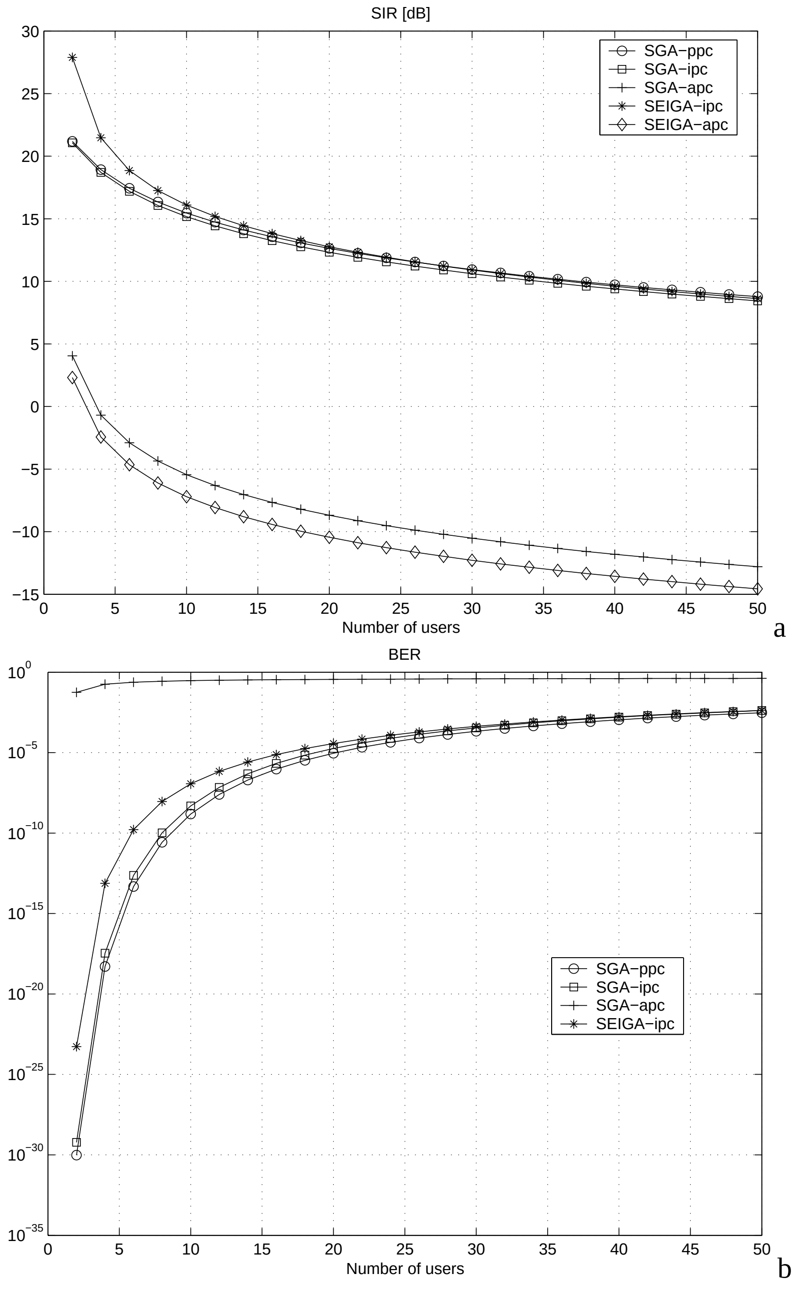


Figure A.5: SIR (a) and BER (b) over a non-fading channel with perfect (ppc), imperfect (ipc) and absent power control (apc); $G_p = 128$.

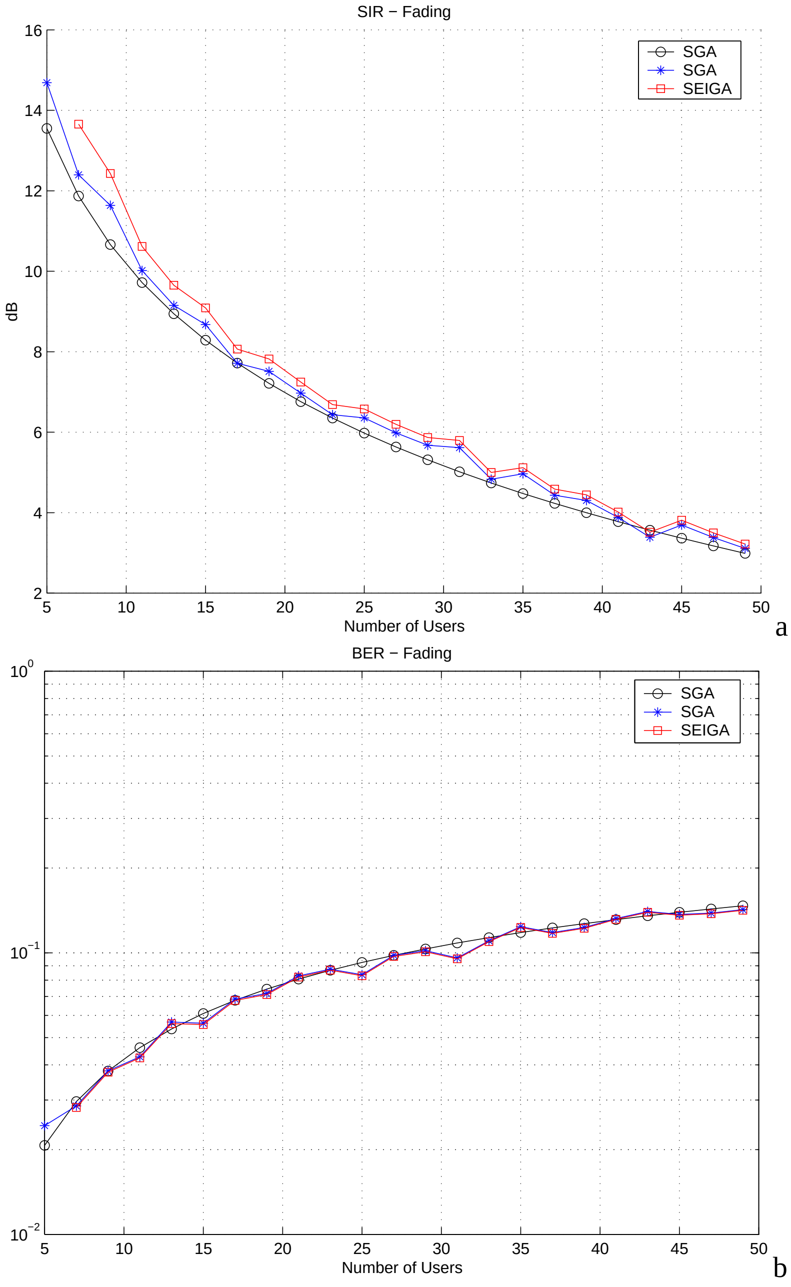


Figure A.6: SIR (a) and BER (b) over a fading channel with perfect power control; $G_p = 32$.

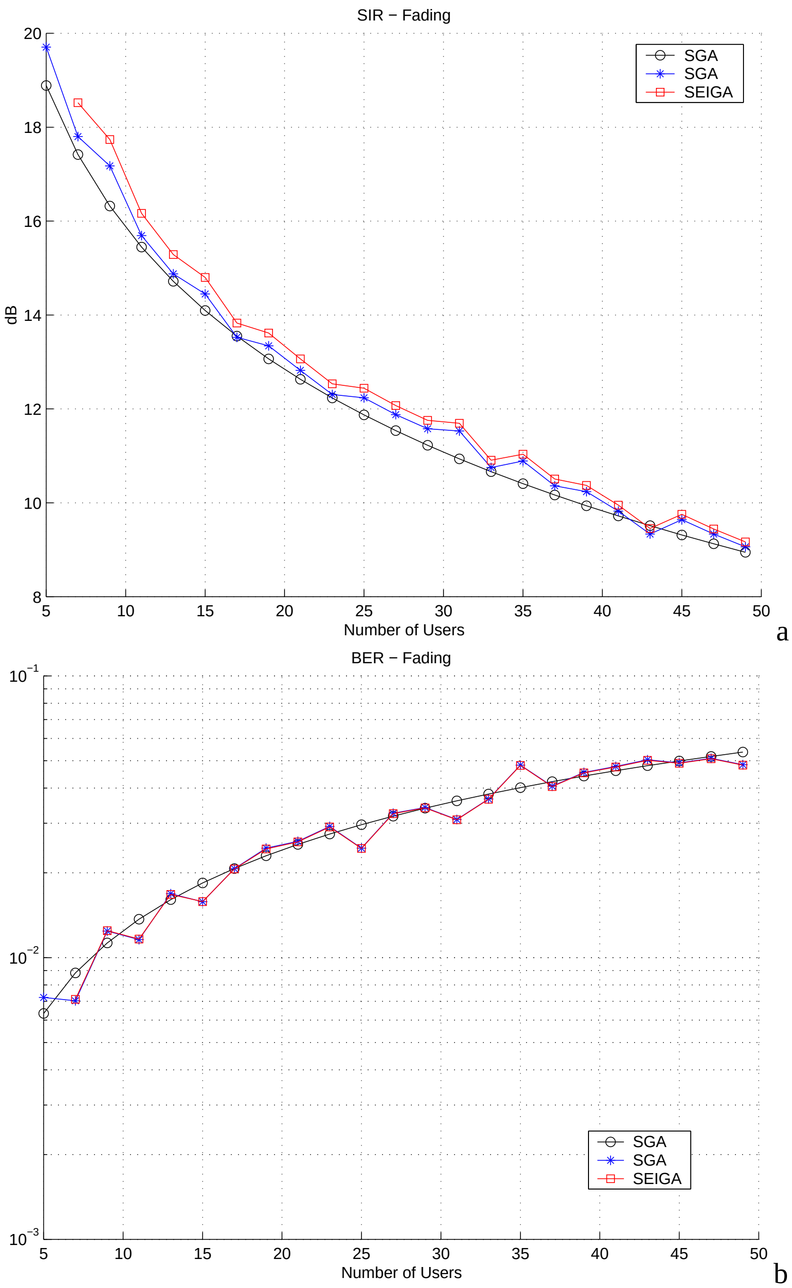


Figure A.7: SIR (a) and BER (b) over a fading channel with perfect power control; $G_p = 128$.

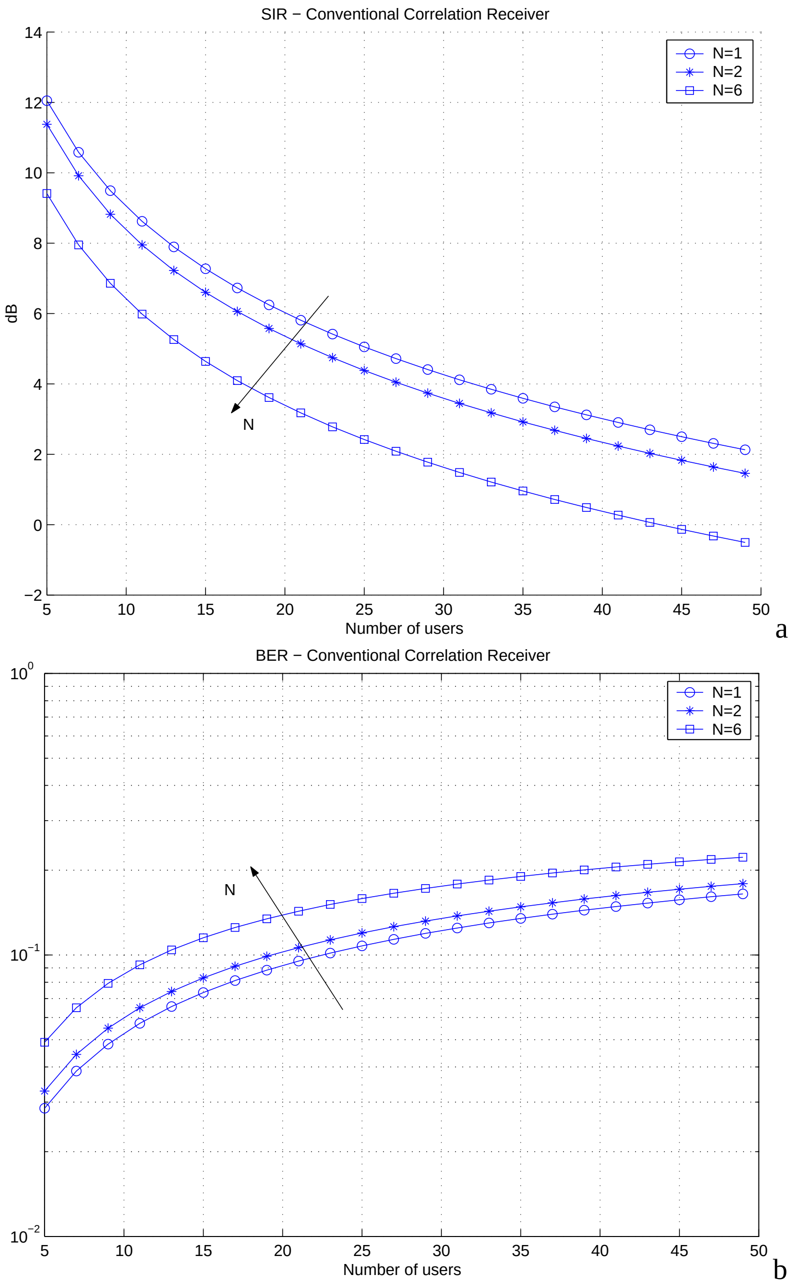


Figure A.8: SIR (a) and BER (b) over a fading channel with perfect power control, as a function of the number of interfering cells N. Conventional correlation receiver, $G_p = 128$, number of multipath $M = 4$.

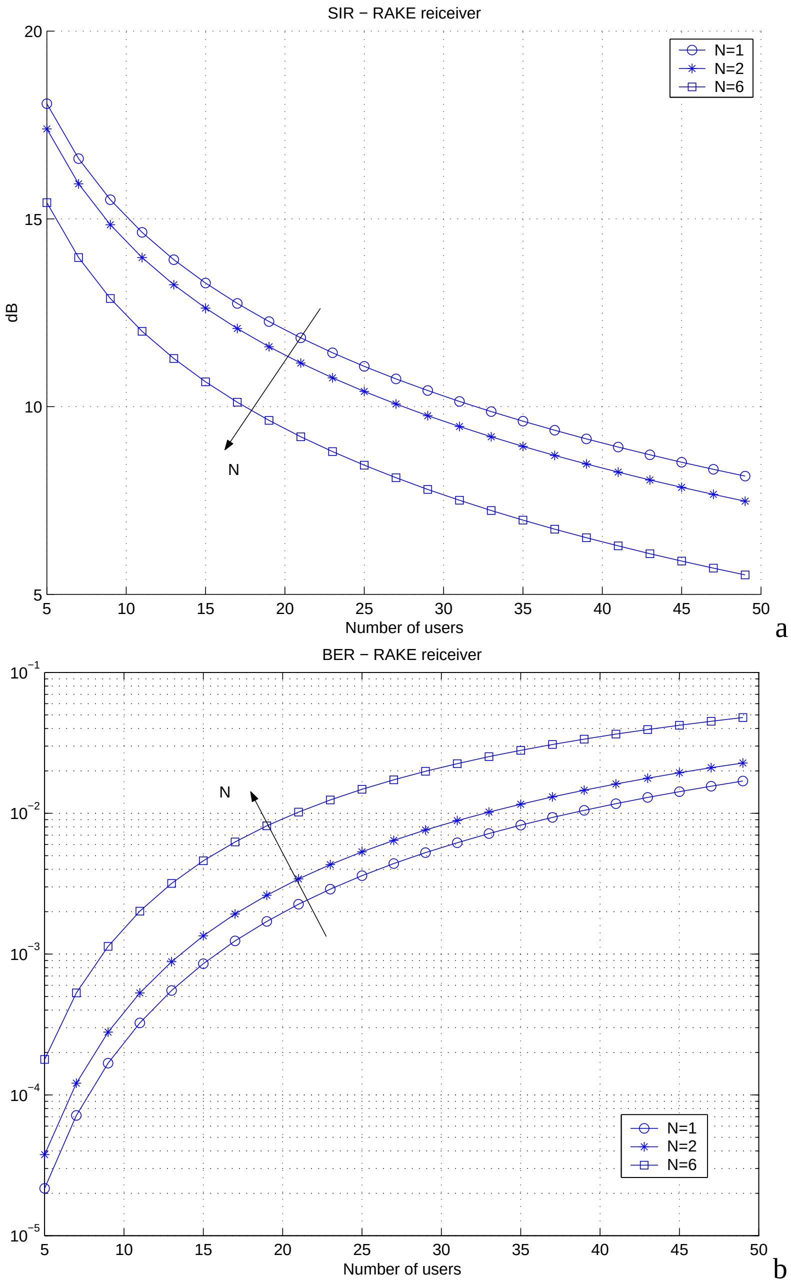
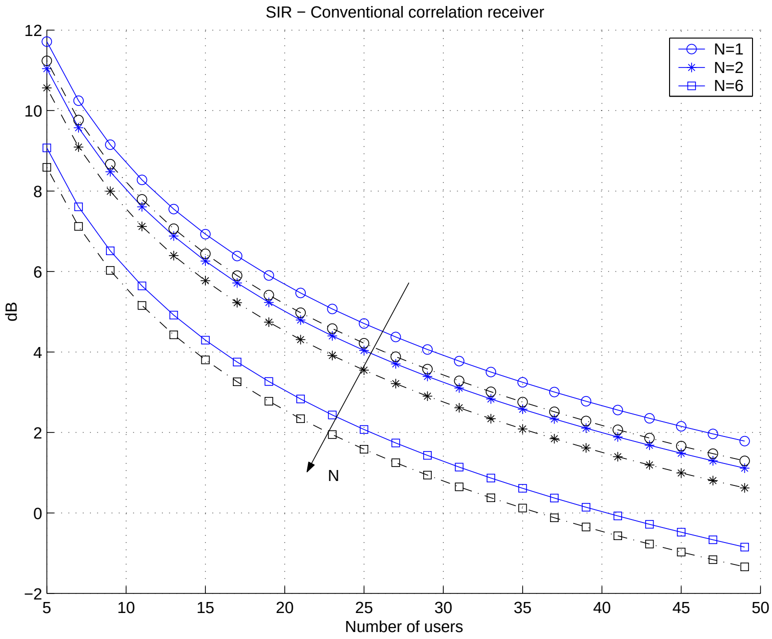


Figure A.9: SIR (a) and BER (b) over a fading channel with perfect power control, as a function of the number of interfering cells N. RAKE receiver, $G_p = 128$, number of multipath $M = 4$.



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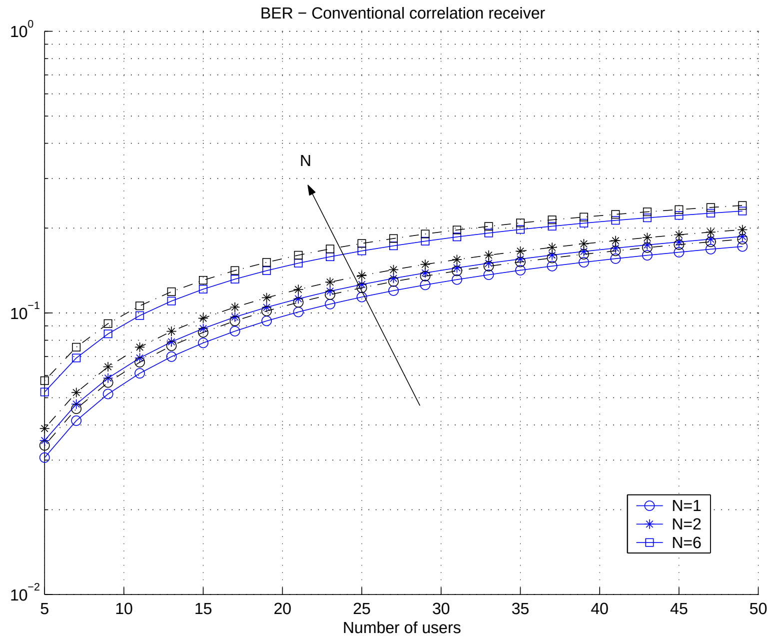

b

Figure A.10: SIR (a) and BER (b) over a fading channel with imperfect power control, as a function of the number of interfering cells N; $\sigma^2 = 1$, $G_p = 128$, number of multipath $M = 4$. Conventional correlation receiver; $k = 0.5$ in solid line; $k = 0.8$ in dash-dot line.

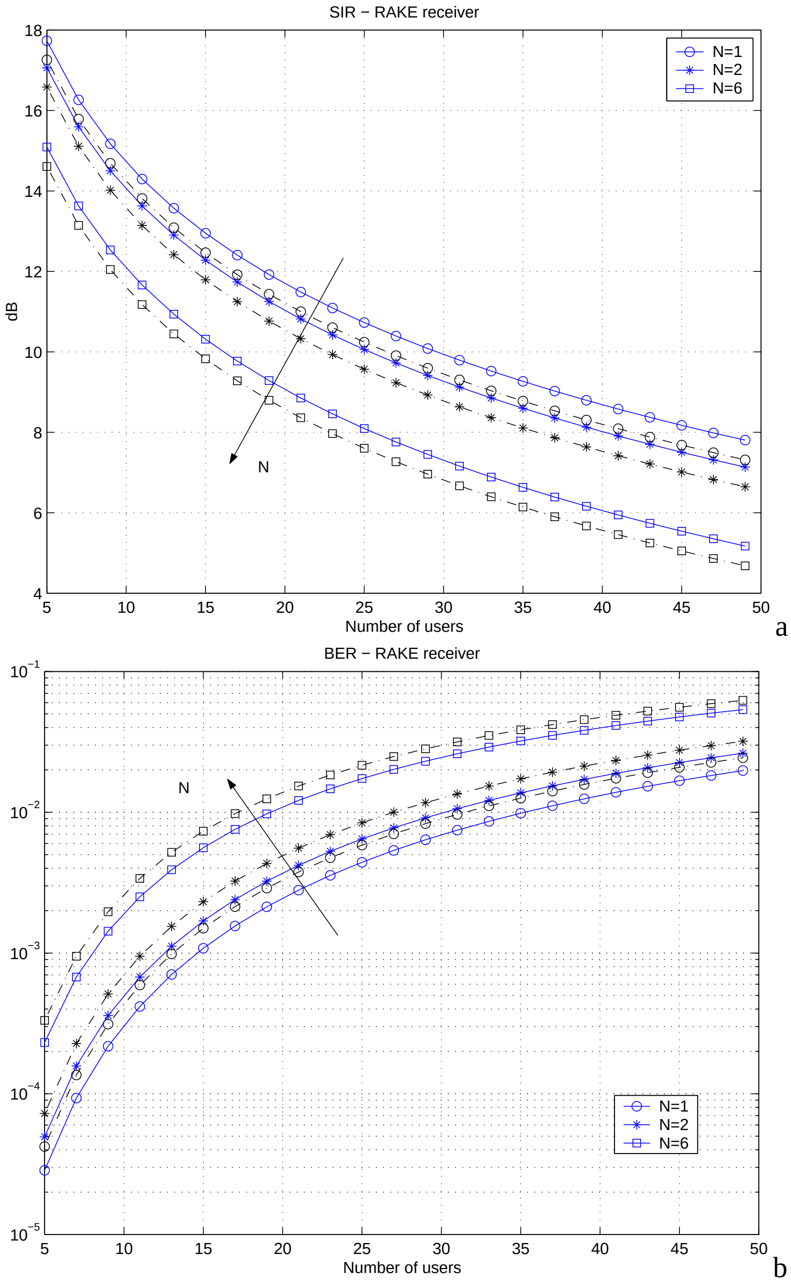

Figure A.11: SIR (a) and BER (b) over a fading channel with imperfect power control, as a function of the number of interfering cells N; $\sigma^2 = 1$, $G_p = 128$, number of multipath $M = 4$. RAKE receiver; $k = 0.5$ in solid line; $k = 0.8$ in dash-dot line.

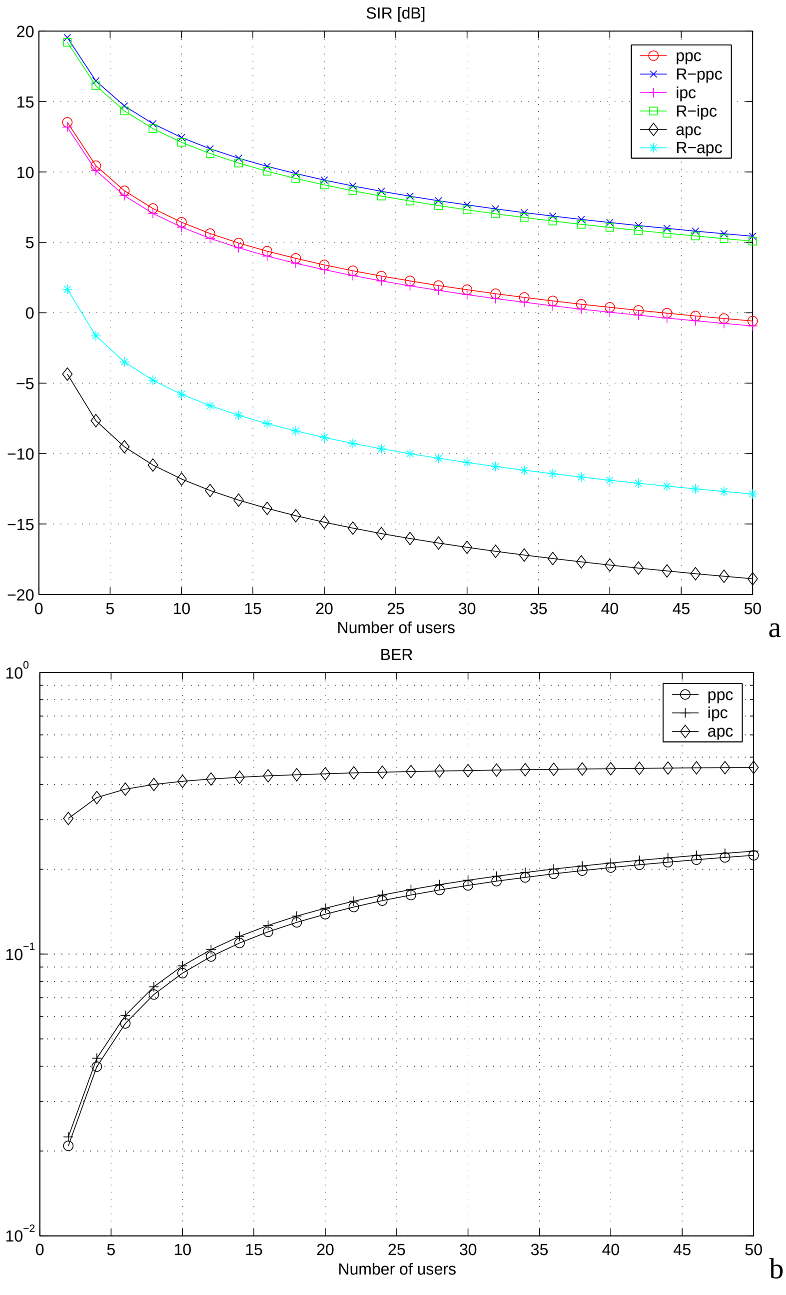

Figure A.12: SIR (a) and BER (b) over a fading channel with perfect (ppc), imperfect (ipc) and no power control (apc), with conventional and RAKE receiver (R); $G_p = 128$.

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