Techical Report
Analysis of Route Stability in MANETs

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Abstract

A fundamental issue arising in mobile ad-hoc networks (MANETs) is the selection of the optimal path between any two nodes. A method that has been advocated to improve routing efficiency is to select the most stable path so as to reduce the latency and the overhead due to route reconstruction. In this work we study the stability of a routing path, which is subject to link failures caused by node mobility, and we consider as metrics of interest the duration and the availability of a path. We first derive some results in the case of a bidimensional Brownian motion, which allows for an analytical study. Although being in some cases unable to represent node mobility in real networks, we show that the analysis obtained under the Brownian motion come in handy when more complex and realistic mobility models are considered. In particular, we present the case of the Random Direction model, and discuss how our findings apply to this scenario. Moreover, using the results on path duration and availability, we show how to determine the optimal path in terms of route stability, under both the Brownian and the Random Direction mobility models.

I. Introduction

Mobile wireless networks are receiving an increasing interest, due to the possibility of ubiquitous communications they offer. In particular, mobile ad hoc networks (MANETs) enable users to maintain connectivity to the fixed network or exchange information when no infrastructure, such as a base station or an access point, are available. This is achieved through multihop communications, which allow a communication node to reach far away destinations by using intermediate nodes as relays.

Traffic relaying in MANETs, however, is a difficult task. Node mobility, signal interference and power outages make the network topology frequently change. As a consequence, the links along a route may fail, and an alternate path must be found. To avoid the degradation of the system performance, several solutions for route creation and maintenance have been proposed in the literature, taking into account various metrics of interest. A method that has been advocated to improve routing efficiency is to select the most stable path [1], [2], [3], [4] so as to avoid packet losses and limit the latency and overhead due to route reconstruction. Specifically, an algorithm should be able to select a route based on some knowledge of the nodes motion and on a probability model of the future availability of the route. Furthermore, if an estimate of the route duration is available, route disruption can be avoided by creating an alternative path before the current one breaks [4].

From the considerations above, it follows that route duration and availability are fundamental issues in MANETs. In this work, we focus on the stability of a routing path, which is subject to link failures caused by node mobility, and consider as metrics of interest the route duration and route availability, hereinafter also called *path duration* and *path availability*, respectively. We define the route duration as the time interval from when the route is established until one of the links along the route becomes unavailable, while we say that a path is available at a given time instant t when all links along the path are active at time t.

It is clear that the duration and the availability of a routing path strongly depend on the mobility pattern of the network nodes, and one would like to be able to evaluate these metrics in presence of various mobility models. Unfortunately, a study of route stability is extremely difficult even under simple mobility patterns, such as the random waypoint model [5]. (Please see Section II for a detailed discussion on related work and previous results.) Here we focus on bidimensional random mobility models [7], and we start by considering nodes moving according to independent Brownian motions. Although in some cases unable to represent node mobility in real networks, the Brownian model has several nice properties: (i) the relative motion between two nodes is still Brownian; (ii) it is possible to mathematically compute the distribution of the distance between two mobile nodes under several network scenarios of interest. Such properties allow for a theoretical analysis, whose results come in handy when more complex and realistic mobility models are considered. To show this, we present the case of the Random Direction (RD) mobility model [6], [8], and discuss how our findings apply to this scenario.

Our main contributions are as follows.

- We derive the link availability and link duration probability under a bidimensional Brownian motion, being the initial nodes distance any positive value shorter than the transmission range (Section IV). We highlight that, under this general assumption, the analysis becomes more complex than in the case where the nodes start moving from the same location [9].
- In the case of the Random Direction mobility model, we first derive an expression for the transform of the node spatial distribution, which can be numerically antitransformed, and a closed expression for the temporal evolution of the distribution moments (Section V). Then, we obtain an exact expression for the link availability, as well as a simpler,

approximate expression based on the observation that after some time the node spatial distribution can be represented by a normal distribution (Section V-B). This result turns out to be very useful while studying the path stability under the Random Direction model.

- Based on the results above, we study the stability of multihop paths, in the case of both Brownian and Random Direction motion. We discuss the validity of the link independence assumption, which is widely used, and compare it with a refined assumption, which accounts for link correlation (Section VI). We observe that the link independence assumption provides sufficiently accurate results in many cases of practical interest.
- Finally, we show how our analysis can be exploited to improve the efficiency of traffic routing in MANETs. In particular, we show how to select the optimal path in terms of route stability, and how to determine the optimal number of hops under the Brownian and Random Direction models (Section VII).

II. Related Work

Several previous works have addressed the problem of link and route stability. Routing protocols accounting for route stability while selecting the source-destination path can be found in [1], [2], [3], [4], just to name a few. In particular, the work in [4] considers nodes moving along non-random patterns and exploits some knowledge of the nodes motion to predict the path duration. This information is then used for route selection as well as for providing another path before the current one breaks.

Studies on link and path availability and duration are presented in [9], [10], [11], [12], [13], [14], [15], [16], [17]. In [9], a deterministic, a partially deterministic and a Brownian motion, where nodes start moving from the same location, are considered. Note that our analysis is instead carried out under general initial conditions, which makes the analysis more complex. In [10], [11], [12], [13] nodes move with random direction and at random velocity, but both direction and velocity are kept constant over time so that the link [10], [11] and the path [12], [13] duration can be analyzed using geometric observations.

Most studies analyzing path duration make the assumption of independent behavior of the links along the path. This assumption is removed in [14], where the *joint* probability distribution of the path duration is derived, but only for the case of a discrete random walk model. In our work, we discuss the validity of the assumption on link independence and find that it works fairly well in several cases of practical interest.

Relevant to our work is also the study in [15], where the impact of mobility on message latency is analyzed in the case in which nodes act as relays. The authors consider a simple linear topology where nodes move along adjacent segments with reflecting boundaries according to unidimensional Brownian motion. Transfer of messages between adjacent nodes can take place only when nodes are in contact, i.e., they are within the radio range of each other. By assuming that the message transmission is instantaneous, they evaluate the expected latency of a message transferred from the first to the last node of a sequence. Note however that, since the focus in [15] is on traffic relaying rather than on link and path stability, the analysis in [15] aims essentially at the evaluation of the expected first contact time between adjacent nodes.

Finally, in [16] it is shown via simulation that the probability distribution of the path duration is accurately approximated by an exponential distribution when the number of hops along the path is sufficiently large, no matter which mobility model is considered. This observation is confirmed by the analytical study in [17]. The work in [17] exploits Palm's calculus by assuming that links along a path are independent and that steady state is reached. Even if the results in [16], [17] have a significant theoretical value, their practical relevance appears to be limited by the following two facts: (i) they hold only for paths including a large number of hops; (ii) the parameter of the exponential distribution has to be empirically obtained from the map layout, node density and other detailed parameters of the mobility scenario.

III. Assumptions and Definitions

While studying path duration and availability in MANETs, we make the following assumptions.

(i) The network comprises homogeneous nodes moving over a bidimensional area; in particular, all nodes have a common transmission range, R, and have the same mobility pattern.
(ii) Nodes move independently of each other.
(iii) A free space propagation model is considered, i.e., the received signal only depends on its distance from the transmitter.
(iv) Communication links are bidirectional.

Let us consider two generic nodes, A and B, and let $\mathbf{X}_A(t)$ and $\mathbf{X}_B(t)$ be their positions, respectively, at time t. We define the distance between the two nodes at time t as: $d_{A,B}(t) = ||\mathbf{X}_A(t) - \mathbf{X}_B(t)||$. According to assumption *(iii)*, a (bidirectional) communication link between A and B exists if the two nodes are within the transmission range of each other. Then, considering assumption *(i)*, we say that a link between A and B exists at time t if $d_{A,B}(t) < R$.

Let us define the link availability $A_{link}(d_{A,B}(0), t)$ as the probability that the link between nodes A and B is active at time t, given that the initial distance between the two nodes is equal to $d_{A,B}(0)$, $0 \leq d_{A,B}(0) < R$, i.e.,

$$A_{link}(d_{A,B}(0), t) = \mathbb{P}(d_{A,B}(t) < R | d_{A,B}(0)) \tag{1}$$

We define the link duration probability, $D_{link}(d_{A,B}(0),t)$, as the probability that the link between A and B has been uninterruptedly active till time t, given that their initial distance is $d_{A,B}(0)$,
$0 \le d_{A,B}(0) < R$,

$$D_{link}(d_{A,B}(0),t) = \mathbb{P}(\inf\{\tau \text{ s.t. } d_{A,B}(\tau) > R\} > t | d_{A,B}(0)) \tag{2}$$

Now, consider $n+1$ mobile nodes, and let $\mathbf{X}_i(t)$ be the position of node i with $1 \le i \le n+1$ at time t. Assume that $d_{i,i+1}(0) < R$ for $1 \le i \le n$ and let us denote by $\mathbf{d}_0 = \{d_{i,i+1}(0)\}$ the vector of initial nodes distances $d_{i,i+1}(0)$, $1 \le i \le n$. Then, consider a path of n hops, obtained by visiting the $n+1$ nodes in sequence: $1 \to 2 \to 3 \to \ldots \to n+1$. The path availability at time t is defined as:

$$A_{path}(\mathbf{d}_0,t) = \mathbb{P}(d_{i,i+1}(t) < R, \forall i \le n | \mathbf{d}_0) \tag{3}$$

while the path duration probability $D_{path}(\mathbf{d}_0,t)$ is the probability that the path has been uninterruptedly active till time t:

$$D_{path}(\mathbf{d}_0,t) = \mathbb{P}(\inf\{\tau \text{ s.t. for some } i, 1 \le i \le n \; d_{i,i+1}(\tau) > R\} > t | \mathbf{d}_0) \tag{4}$$

IV. Link Stability under the Brownian Mobility Model

Consider two generic nodes, A and B, moving over a infinite plane according to independent Brownian motions. We assume Brownian motions with null infinitesimal mean and (infinitesimal) variance equal to σ^2.

Clearly the distance $d_{A,B}(t)$ between A and B depends on the *relative* motion of the two nodes, and in general deriving the distribution of the nodes distance is very difficult. In the case of Brownian motion, however, $d_{A,B}(t)$ evolves over time as a function of the relative motion of A and B, which is again a Brownian motion with null infinitesimal mean and variance equal to the the sum of the variance of each (i.e., equal to $2\sigma^2$). The evolution over time of the relative position of two nodes (i.e., the evolution of node B position in a coordinates system centered in node A), is the unique solution of the the well known bidimensional heat equation over an infinite bidimensional domain with assigned initial condition:

$$\frac{\partial f_d(x,y,t)}{\partial t} = \sigma^2 \nabla^2 f_d(x,y,t) \tag{5}$$

In (5) $f_d(x,y,t)$ represents the relative position distribution at time t, with $f_d(x,y,0)$ being its initial condition, and ∇^2 is the Laplacian operator.

It results (see [18]):

$$f_d(x,y,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K(x-z,y-w,t) f_d(z,w,0)\,dz\,dw \tag{6}$$

where $K(x-z,y-w,t)$ is the Green's function of the heat equation[1]:

$$K(x-z,y-w,t) = \frac{1}{4\pi\sigma^2 t} e^{-\frac{(x-z)^2+(y-w)^2}{4\sigma^2 t}} \tag{7}$$

Now, assuming a deterministic initial condition, let us express the distribution of the relative position in polar coordinates. The link availability at time t, can be computed by integrating the relative position distribution at time t over a circle of radius R:

$$A_{link}(d_{A,B}(0),t) = \int_0^R \int_0^{2\pi} f_d(\rho,\theta,t)\;\rho\,\mathrm{d}\rho\,\mathrm{d}\theta = \int_0^R \frac{1}{2\sigma^2 t} e^{-\frac{\rho^2+d^2_{A,B}(0)}{4\sigma^2 t}} I_0\left(\frac{\rho d_{A,B}(0)}{2\sigma^2 t}\right)\rho\,\mathrm{d}\rho \tag{8}$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind. Note that the distance between A and B is described by a Rice distribution. In the case of unidimensional Brownian motion, the link availability reduces to the following simple expression:

$$A_{link}(d_{A,B}(0),t) = 1 - Q\left(\frac{R+d_{A,B}(0)}{\sqrt{2\sigma^2 t}}\right) - Q\left(\frac{R-d_{A,B}(0)}{\sqrt{2\sigma^2 t}}\right) \tag{9}$$

where $Q(\cdot)$ is the tail of a normal distribution with zero mean and unit variance.

Next, we compute the link duration probability, $D_{link}(d_{A,B}(0),t)$. Recall that (5) describes the evolution of node B position in a system of coordinates centered in node A. Then, $D_{link}(d_{A,B}(0),t)$ can be analytically evaluated by solving (5) over a circular spatial domain of radius R, with absorbing boundary conditions, i.e., by forcing $f_d(x,y,t) = 0 \;\forall x,y$ such that $\sqrt{x^2+y^2} = R$. Equation (5) can be analytically solved in polar coordinates by applying a standard variables separation method. It turns out that the general weak solution of (5) with arbitrary initial condition and the absorbing boundary conditions, is given by:

$$f_{d-bound}(\rho,\theta,t) = \sum_{n=0}^{\infty}\sum_{i=0}^{\infty} (\beta_{ni}\cos(n\theta) + \gamma_{ni}\sin(n\theta)) \cdot J_n\left(z_i\frac{\rho}{R}\right) e^{-(\frac{z_i}{R})^2\sigma^2 t} \tag{10}$$

[1]We notice that $K(x-x_0,y-y_0,t)$ represents the distribution of the relative position of nodes at time t under a deterministic initial condition $f_d(x,y,0) = \delta(x-x_0)\delta(y-y_0)$. In this case the variance of $f_d(x,y,t)$ is $2\sigma^2 t$

J_n is the n-th order Bessel function of first kind, z_i is the $i-$th zero of the considered Bessel function, while the coefficients β_{ni} and γ_{ni} represent the orthonormal projections of the initial condition $f_0(\rho,\theta)$, i.e.,

$$\beta_{ni} = \frac{\int_0^R \int_{-\pi}^{\pi} \cos(n\theta)\, J_n\left(z_i \frac{\rho}{R}\right) f_0(\rho,\theta) \rho\, \mathrm{d}\rho\, \mathrm{d}\theta}{\int_{-\pi}^{\pi} \cos^2(n\theta)\, d\theta \int_0^R J_n^2\left(z_i \frac{\rho}{R}\right) \rho\, \mathrm{d}\rho}$$

$$\gamma_{ni} = \frac{\int_0^R \int_{-\pi}^{\pi} \sin(n\theta)\, J_n\left(z_i \frac{\rho}{R}\right) f_0(\rho,\theta) \rho\, \mathrm{d}\rho\, \mathrm{d}\theta}{\int_{-\pi}^{\pi} \sin^2(n\theta)\, d\theta \int_0^R J_n^2\left(z_i \frac{\rho}{R}\right) \rho\, \mathrm{d}\rho}$$

The link duration probability $D_{link}(d_{A,B}(0),t)$ can be now computed by integrating the above solution in the disk of radius R:

$$\begin{aligned} D_{link}(d_{A,B}(0),t) &= \int_0^R \int_0^{2\pi} f_{d-bound}(\rho,\theta,t)\ \rho\, \mathrm{d}\rho\, \mathrm{d}\theta \\ &= \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} e^{-(\frac{z_i}{R})^2 \sigma^2 t} \Big[\beta_{ni} \int_0^R \int_0^{2\pi} \cos(n\theta)\ J_n\left(z_i \frac{\rho}{R}\right) \rho\, \mathrm{d}\rho\, \mathrm{d}\theta + \gamma_{ni} \int_0^R \int_0^{2\pi} \sin(n\theta)\ J_n\left(z_i \frac{\rho}{R}\right) \rho\, \mathrm{d}\rho\, \mathrm{d}\theta \Big] \\ &= 2\pi \sum_{i=0}^{\infty} \beta_{0i} e^{-(\frac{z_i}{R})^2 \sigma^2 t} \int_0^R J_0\left(z_i \frac{\rho}{R}\right) \rho\, \mathrm{d}\rho \end{aligned} \quad (11)$$

where Lebesgue dominated convergence theorem was invoked to commute series and integral operators.

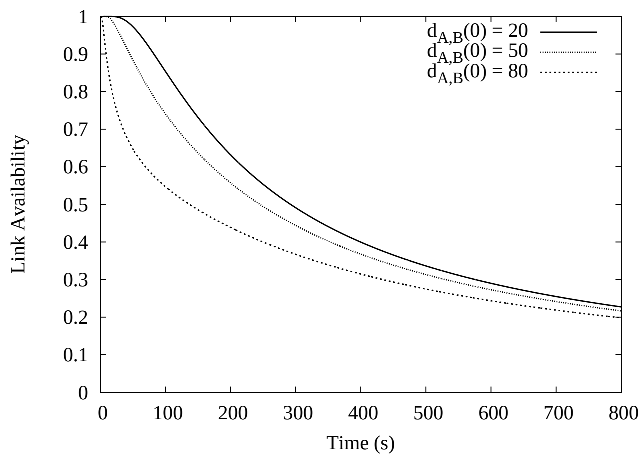

Fig. 1. Link availability as a function of time, for different values of the initial distance between the nodes forming the link

We conclude this section presenting some numerical results obtained in the case of a bidimensional Brownian motion with $\sigma^2 = 12$, and assuming a transmission range $R = 100$ m.

Figure 1 reports the link availability $A_{link}(d_{A,B}(0),t)$ as a function of time, for three different values of $d_{A,B}(0)$. We observe that, at the beginning, the link availability is quite sensitive to the initial distance between the nodes forming the link. As time goes to infinity, instead, the impact of the initial condition tends to vanish. Indeed, when t becomes very large, the integral in (8) can be approximated by $\frac{R^2}{4\sigma^2 t}$, independently of $d_{A,B}(0)$. Hence, the link availability behaves, asymptotically, as $1/t$.

Figure 2 presents the link duration $D_{link}(d_{A,B}(0),t)$ as a function of time, in the same scenario as considered in Figure 1. As expected, the initial distance plays a crucial role: when $d_{A,B}(0)$ is close to the transmission range (e.g., $d_{A,B}(0) = 80$ m), the link duration decreases dramatically, much faster than in the case where the two nodes are initially close to each other (e.g., $d_{A,B}(0) = 20$ m). In the insert of Figure 2 we show the same curves plotted on a log scale. We observe that all curves exhibit, after a short time, an exponential decay with the same parameter, irrespective of the initial condition. Indeed, when time goes to infinity, the dominating term in (11) is the exponential function associated to the first zero of the Bessel function J_0, which is at $z^* \simeq 2.405$ (i.e., $e^{-(\frac{z^*}{R})^2 \sigma^2 t}$). In the plot, we present the curve corresponding to the only term $e^{-(\frac{z^*}{R})^2 \sigma^2 t}$ taken from (11), labeled as 'exp decay' (notice that it does not depend on the initial condition). This curve shows that the parameter of the exponential decay is indeed given by $(z^*/R)^2 \sigma^2$.

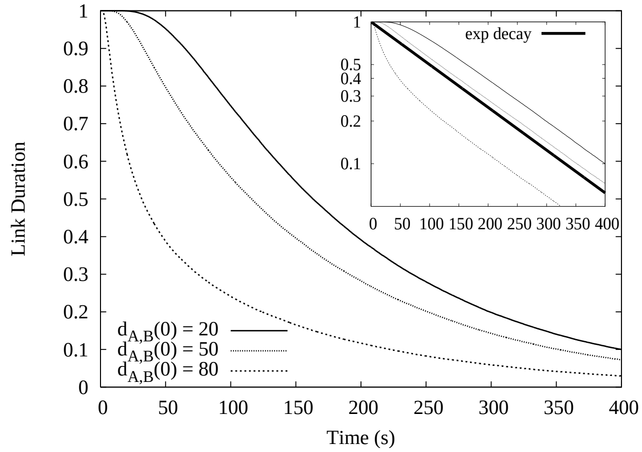

Fig. 2. Link duration as a function of time, for different values of the initial distance between the nodes forming the link

V. Link Stability under the Random Direction Model

In this section we consider a more realistic mobility model, the so-called Random Direction model (RD) [6]. Each node alternates periods of movement (*move phase*) to periods during which it pauses (*pause phase*), and we assume that the duration of the phases are randomly distributed. At the beginning of each *move* phase, a node independently selects its new direction and speed of movement; speed and direction are maintained constant for the whole duration of the node's *move* phase [6].

Under the RD model, the time evolution of the position of a node, either moving or pausing, can be described through a system of partial differential equations (PDE's) [19]. In [19], by applying the methodology of variables separations, it is shown that under mild conditions a (weak) solution of the above equations over a rectangular area can be found. Here, instead, we study the dynamics of nodes moving according to an RD model over an infinite bidimensional domain. We obtain a closed expression for the general (weak) solution of the RD equations in [19] in the frequency domain (i.e., the moment generating function), under the assumption that move and pause times are exponentially distributed with parameter μ and λ, respectively. Even if a direct analytical antitransformation of the obtained moment generating function appears to be prohibitive, closed expressions for the moments of the spatial probability density function (pdf) can be easily derived. By using the node spatial distribution, we write an exact expression for the link availability, and then propose a simple approximation to evaluate this metric based on the second moment of the spatial distribution, which provides satisfactory results.

A. Node spatial distribution

Let $\mathbf{x} = (x_1, x_2)$ be the position of the mobile node and $\mathbf{v} = (v_1, v_2)$ its current speed vector (each component represents the node's position/speed along the corresponding dimension). Then the dynamics of the node movement can be described in terms of a Markov Process over a general space state [20], in which the instantaneous node state is characterized by: i) the phase $P(t) \in \mathcal{P} = \{\textit{move}, \textit{pause}\}$; ii) the instantaneous position $\mathbf{X}(t)$; iii) the current speed $\mathbf{V}(t)$ (when $P(t) = \textit{move}$).

Let $m(\mathbf{x}, \mathbf{v}, t)$ be the pdf at time t over the state space $(\mathbf{x}, \mathbf{v})$ for the node in the *move* phase:

$$m(\mathbf{x}, \mathbf{v}, t) \triangleq \frac{\partial^2 \mathbb{P}(P(t) = \textit{move}, \mathbf{X}(t) \le \mathbf{x}, \mathbf{V}(t) \le \mathbf{v})}{\partial x \partial v}$$

Let $p(x, t)$ be the pdf at time t over the state space $\mathbf{x}$ for the node in the *pause* phase:

$$p(\mathbf{x}, t) \triangleq \frac{\partial \mathbb{P}(P(t) = \textit{pause}, \mathbf{X}(t) \le \mathbf{x})}{\partial x}$$

Also, let us assume that at the beginning of each *move* phase, a node selects a speed from the generic distribution $f_V(\mathbf{v})$, with the absolute speed value being upper bounded by a constant $V_{\max}$. This is a reasonable assumption for all cases of practical interest.

The Chapman-Kolmogorov equations of the system are as follows:

$$\frac{\partial m(\mathbf{x}, \mathbf{v}, t)}{\partial t} = -\mathbf{v} \cdot \nabla_{\mathbf{x}} m(\mathbf{x}, \mathbf{v}, t) + \lambda f_V(\mathbf{v}) p(\mathbf{x}) - \mu\, m(\mathbf{x}, \mathbf{v}, t) \tag{12}$$

$$\frac{\partial p(\mathbf{x}, t)}{\partial t} = -\lambda p(\mathbf{x}, t) + \mu \int m(\mathbf{x}, \mathbf{v}, t)\, \mathrm{d}\mathbf{v} \tag{13}$$

being $\mathbf{v} \cdot \nabla_{\mathbf{x}} m(\mathbf{x}, \mathbf{v}, t)$ the inner product between $\mathbf{v}$ and $\nabla_{\mathbf{x}} m(\mathbf{x}, \mathbf{v}, t)$. Since the solution of the above equations lies in L^1 (space of absolutely summable functions) at any time, we can apply a double Fourier transform over the space coordinates, and a unilateral time Laplace transform. We obtain:

$$s\ m(\mathbf{k}, \mathbf{v}, s) - m_0(\mathbf{k}, \mathbf{v}) = -\mathbf{v} \cdot j\mathbf{k} m(\mathbf{k}, \mathbf{v}, s) + \lambda f_V(\mathbf{v}) p(\mathbf{k}) - \mu\, m(\mathbf{k}, \mathbf{v}, s) \tag{14}$$

$$sp(\mathbf{k}, s) - p_0(\mathbf{k}, \mathbf{v}) = -\lambda p(\mathbf{k}, t) + \mu \int m(\mathbf{k}, \mathbf{v}, s)\, \mathrm{d}\mathbf{v} \tag{15}$$

where:

$$\begin{aligned} m(\mathbf{k}, \mathbf{v}, s) &= \int_{\mathbf{x}} \int_{t \geq 0} m(\mathbf{x}, \mathbf{v}, t) e^{-st - j\mathbf{k}\cdot\mathbf{x}} dt d\mathbf{x} \\ p(\mathbf{k}, s) &= \int_{\mathbf{x}} \int_{t \geq 0} p(\mathbf{x}, t) e^{-st - j\mathbf{k}\cdot\mathbf{x}} dt d\mathbf{x} \\ m_0(\mathbf{k}, \mathbf{v}) &= \int m(\mathbf{x}, \mathbf{v}, 0) e^{-j\mathbf{k}\cdot\mathbf{x}} d\mathbf{x} \\ p_0(\mathbf{k}) &= \int p(\mathbf{x}, 0) e^{-j\mathbf{k}\cdot\mathbf{x}} d\mathbf{x} \end{aligned}$$

After some calculations, defining $M(\mathbf{k}, s) = \int m(\mathbf{k}, \mathbf{v}, s) d\mathbf{v}$, we have:

$$M(\mathbf{k}, s) = \frac{\int \frac{\lambda f_V(\mathbf{v}) p_0(\mathbf{k})}{(s+\lambda)(s+\mu+j\mathbf{k}\cdot\mathbf{v})} d\mathbf{v} + \int \frac{m_0(\mathbf{k}, \mathbf{v})}{s+\mu+j\mathbf{k}\cdot\mathbf{v}} d\mathbf{v}}{1 - \int \frac{\lambda\mu f_V(\mathbf{v})}{(s+\lambda)(s+\mu+j\mathbf{k}\cdot\mathbf{v})} d\mathbf{v}} \tag{16}$$

and

$$p(\mathbf{k}, s) = \frac{\mu M(\mathbf{k}, s) + p_0(\mathbf{k})}{s + \lambda} \tag{17}$$

Unfortunately we are unable to analytical antitransform (16); moreover some care is required even to numerical antitransform (16) and (17), due to the presence of a singularity in the origin. However, by manipulating (16) and (17) at the same time, we can gather some important insights on the mobile dynamics, and obtain a different expression for $M(\mathbf{k}, s)$ and $p(\mathbf{k}, s)$ that can be easily antitransformed numerically (please see Appendix for further details). Furthermore, closed form expressions for the temporal evolution of the moments of the spatial distribution can be easily obtained. Let $Y^l(t)$ be the l-th spatial central moment of the distribution at time t, and let $Y^l(s)$ be its Laplace transform. We have:

$$\begin{aligned} Y^l(t) &= \sum_{i=1}^{2} \Big[\int_{\mathbf{x}} \int_{\mathbf{v}} (x_i)^l m(\mathbf{x}, \mathbf{v}, t) d\mathbf{v} d\mathbf{x} + \int (x_i)^l p(\mathbf{x}, t)\ d\mathbf{x} \Big] \\ Y^l(s) &= j^l \sum_{i=1}^{2} \frac{\partial^l}{\partial^l k_i} [M(\mathbf{k}, s) + p(\mathbf{k}, s)]|_{\mathbf{k}=0} \end{aligned}$$

Since $Y^l(s)$ is a rational expression in s, for every l a direct antitransformation can be obtained. In particular, we report here the formula for the variance when the node starts at $t = 0$ in $\mathbf{x} = 0$[2]:

$$\sigma^2(t) = Y^2(t) = 2\frac{\lambda \sigma_V^2}{\mu^2(\mu + \lambda)}(\mu t + e^{-\mu t} - 1) \tag{18}$$

where σ_V^2 is the variance of the node speed distribution $f_V(\mathbf{v})$.

The variance as a function of time and for different values of λ and μ (i.e., the *move* and *pause* phases duration) is shown in Figure 3, for $\sigma_V^2 = 1$ and $\lambda = \mu$. Note that, as time increases, all curves in the plot tend to exhibit a linear behavior since in (18) μt becomes the dominant term. For long durations of the *move* and *pause* phases (i.e., small values of $\lambda = \mu$), the variance increases and the linear behavior appears for larger values of t.

B. Link availability

We now derive an exact expression for the link availability at time t, by using the spatial pdf of nodes moving according to the RD model.

Consider two nodes A and B. Let $M_A(\mathbf{x}, t) = \int m_A(\mathbf{x}, \mathbf{v}, t) d\mathbf{v}$ and $p_A(\mathbf{x}, t)$ be the space distributions of A at time t in the *move* and *pause* phases, respectively, and let $M_B(\mathbf{x}, t) = \int m_B(\mathbf{x}, \mathbf{v}, t) d\mathbf{v}$ and $p_B(\mathbf{x}, t)$ be the space distributions of B at time t in the *move* and *pause* pauses, respectively. The link availability between the two nodes can be expressed as follows:

$$A_{link}(d_{A,B}(0), t) = \int_{\mathbf{x}_A} \int_{\mathbf{x}_B} [M_A(\mathbf{x}_A, t) + p_A(\mathbf{x}_A, t)] \cdot [M_B(\mathbf{x}_B, t) + p_B(\mathbf{x}_B, t)] \mathbb{1}_{||\mathbf{x}_A - \mathbf{x}_B|| < R}\ d\mathbf{x}_A d\mathbf{x}_B \tag{19}$$

[2] The expression of the variance has been obtained considering $\mathbb{P}(P(0) = \mathit{move} = \frac{\lambda}{\lambda+\mu}$ and the node speed distribution $f_V(\mathbf{v})$. Similar expressions can be obtained for different initial conditions

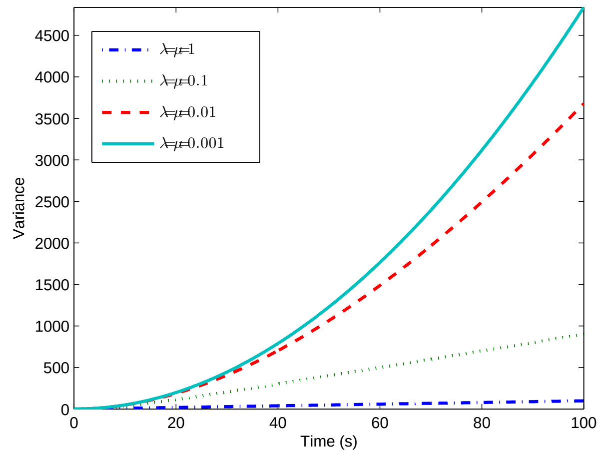

Fig. 3. Variance of the node spatial distribution, when the node starts at $t = 0$ in $\mathbf{x} = 0$

The above expression can be numerically evaluated, however the solution is computationally intensive. Indeed, it requires first to numerically evaluate the spatial distribution of nodes A and B at time t through a tridimensional antitransformation (two spatial dimensions and one temporal dimension), then to numerically compute the integral in (19) over a four-dimensional domain. For these reasons, we propose an approximate methodology for the evaluation of $A_{link}(d_{A,B}(0), t)$, which relies on approximating the spatial distribution of each node with a normal distribution with the same mean and variance. This approximation is justified by the fact that, at time t the total movement of a node with respect to its initial position is essentially determined by the vectorial sum of the elementary movements associated to its accomplished *move* phases. Being elementary movements independent and identically distributed, the central limit theorem can be invoked to claim that the marginal spatial distribution of the nodes tends to be a normal distribution for sufficiently large t.

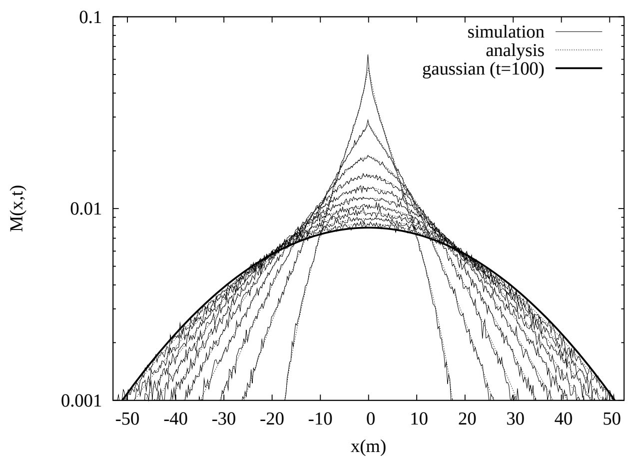

Fig. 4. Spatial distribution in the *move* phase sampled at $t = 20, 40, 60, \ldots, 200$ s

As an example, in Figure 4 we show the numerical antitransform of $M(\mathbf{k}, s)$ in (16), in case of a unidimensional RD model in which *move* and *pause* times have an average of 5 s ($\lambda = \mu = 1/5$), while the speed is uniformly distributed in $[-2, 2]$ m/s. The plot presents on a log scale the spatial distribution $M(\mathbf{x}, t)$ in the *move* phase of a node, which starts at time $t = 0$ in $\mathbf{x} = 0$ in the *pause* phase. The spatial distribution is sampled every 10 s, i.e., the average duration of a cycle including one *move* phase and one *pause* phase. We compare simulation results with the numerical antitransform obtained following the methodology described in Appendix . We observe that, after a few cycles, the spatial distribution takes a bell shape. On the same plot we also reported, for $t = 100$ s only, a normal distribution having the same variance of the node's spatial distribution. We notice that, after 10 cycles, the normal approximation is indeed very good.

Under the previous approximation and denoting by $\mathcal{N}(\mu, \sigma^2, \mathbf{x})$ a normal distribution with average μ and variance σ^2, we have:

$$A_{link}(d_{A,B}(0),t) = \int_{\mathbf{x}_A}\int_{\mathbf{x}_B} \mathcal{N}(\mathbf{x}_0^A,\sigma^2(t),\mathbf{x}_A)\mathcal{N}(\mathbf{x}_0^B,\sigma^2(t),\mathbf{x}_B)\mathbb{1}_{||\mathbf{x}_A-\mathbf{x}_B||<R}\ d\mathbf{x}_A d\mathbf{x}_B$$
$$= \int_{||\mathbf{x}||<R} \mathcal{N}(\mathbf{x}_0^A-\mathbf{x}_0^B,2\sigma^2(t),\mathbf{x})\ d\mathbf{x} = \frac{1}{2\sigma^2(t)}\int_0^R e^{-\frac{\rho^2+d^2_{A,B}(0)}{4\sigma^2(t)}} I_0\left(\frac{\rho d_{A,B}(0)}{2\sigma^2(t)}\right)\ \rho d\rho \quad (20)$$

where we have exploited the fact that the relative position of nodes A and B, $\mathbf{X}_A(t)-\mathbf{X}_B(t)$, is still normally distributed, being the individual position of nodes A and B described by independent normal random variables.

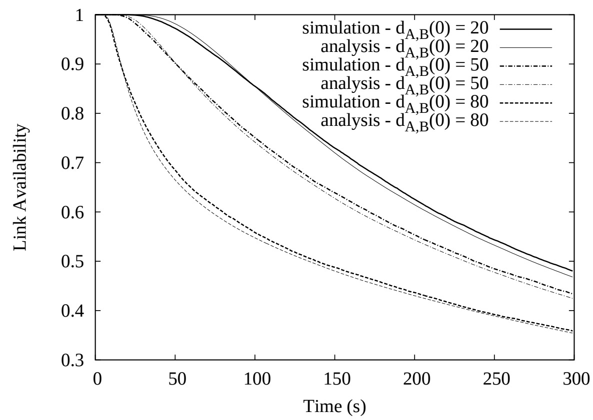

Fig. 5. Link availability as a function of time in case of the RD model, for different values of the initial distance between the nodes forming the link. Analytical results have been obtained using our approximation in (20)

Figure 5 presents the link availability computed through the approximation as in (20), for a bidimensional RD model. The *move* and *pause* times have an average of 10 s ($\lambda = \mu = 1/10$), the speed is uniformly distributed in $[-2, 2]$ m/s, and the transmission range is $R = 100$ m. We consider three different values of $d_{A,B}(0)$, similarly to what we have done in Figure 1 for the Brownian motion. Comparing with simulation results, we observe that our approximation gives excellent results in all cases; in particular, even when the normal approximation is not accurate (i.e., for small values of t) simulation and analytical results are very close. Hence, in the following we use (20) to compute the link availability of the RD model.

C. *Link duration*

An exact expression for the link duration probability under the RD model appears prohibitive for the following reasons.

- The relative motion between two nodes moving according to independent RD motions, is no longer a RD motion. In principle, the relative motion between nodes A and B can be described in terms of a Markov Process over a general state space [20], and deriving the corresponding Chapman-Kolmogorov equations. However the dimensionality of the state space increases since the instantaneous system state is characterized by: i) the phases of the nodes $P_A(t) \in \mathcal{P} = \{\textit{move}, \textit{pause}\}$ and $P_B(t) \in \mathcal{P} = \{\textit{move}, \textit{pause}\}$; ii) the instantaneous relative position $\mathbf{X}_A(t) - \mathbf{X}_B(t)$; iii) the current speed of the two nodes $\mathbf{V}_A(t)$ and $\mathbf{V}_B(t)$.
- The obtained Chapman-Kolmogorov equations should be solved over a circular spatial domain of radius R, with absorbing boundary conditions. However, the structure of the equations describing the relative motion becomes much more complex when they are expressed in polar coordinates, and any attempt to apply the methodology of separable variables fails.

For these reasons, some approximations are needed to evaluate the link duration for the RD model. A coarse approach would be to plug the instantaneous variance of the spatial distribution (which can be computed exactly by (18)) directly into (11), which was derived under the Brownian motion, in place of $\sigma^2 t$. We are yet to explore the accuracy of this approximation.

VI. Multihop paths

Consider $n+1$ nodes moving according to either a Brownian or a Random Direction motion, and assume that $d_{i,i+1}(0) < R$, $1 \leq i \leq n$. Then, consider the path of n links obtained traversing the $n+1$ nodes in sequence. Since nodes movements are assumed to be independent, processes $\mathbf{X}_i(t)$, $1 \leq i \leq n+1$ are independent as well. It follows that $d_{i,i+1}(t)$ and $d_{k,k+1}(t)$ are independent if $|k-i| > 1$, but lengths of adjacent hops, such as $d_{i,i+1}(t)$ and $d_{i+1,i+2}(t)$, are not independent.

The existing correlation between adjacent hop lengths makes an exact analysis of the path dynamics very difficult. We propose two different approximate methodologies to evaluate $A_{path}(\mathbf{d}_0, t)$ or $D_{path}(\mathbf{d}_0, t)$. For simplicity, in the following we refer to the path availability, but we emphasize that a similar discussion can be done for the path duration.

A simple approximation is to assume that links dynamics are independent. In this case $A_{path}(\mathbf{d}_0, t)$ can be easily expressed in terms of $A_{link}(d_{i,i+1}(0), t)$ as follows:

$$A_{path}(\mathbf{d}_0, t) = \Pi_{i=1}^{n} A_{link}(d_{i,i+1}(0), t) \tag{21}$$

being $A_{link}(d_{i,i+1}(0), t)$ the probability that the link between nodes i and $i+1$ is available at time t.

A more accurate approximation can be obtained in the following way. First, consider the case of a 3-link path:

$$\begin{aligned} A_{path}(\mathbf{d}_0, t) &= \mathbb{P}(d_{1,2}(t) < R, d_{2,3}(t) < R, d_{3,4}(t) < R) \\ &= \mathbb{P}(d_{1,2}(t) < R, d_{3,4}(t) < R | d_{2,3}(t) < R\} \, \mathbb{P}(d_{2,3}(t) < R) \\ &\simeq \mathbb{P}(d_{1,2}(t) < R | d_{2,3}(t) < R) \mathbb{P}(d_{3,4}(t) < R | d_{2,3}(t) < R) \\ &\quad \mathbb{P}(d_{2,3}(t) < R) \\ &= \mathbb{P}(d_{1,2}(t) < R, d_{2,3}(t) < R) \mathbb{P}(d_{3,4}(t) < R | d_{2,3}(t) < R) \end{aligned} \tag{22}$$

where the approximation comes from considering the events $d_{1,2}(t) < R$ and $d_{3,4}(t) < R$ conditionally independent given the event[3] $d_{2,3}(t) < R$.

Iterating the previous approach, we can obtain an approximate expression of $A_{path}(\mathbf{d}_0, t)$ for paths of any length. For example, in the case of a 4-link path:

$$\begin{aligned} A_{path}(\mathbf{d}_0, t) &= \mathbb{P}(d_{1,2}(t) < R, d_{2,3}(t) < R, d_{3,4}(t) < R, d_{4,5}(t) < R) \\ &= \mathbb{P}(d_{1,2}(t) < R, d_{2,3}(t) < R, d_{4,5}(t) < R | d_{3,4}(t) < R) \, \mathbb{P}(d_{3,4}(t) < R) \\ &\simeq \mathbb{P}(d_{1,2}(t) < R, d_{2,3}(t) < R | d_{3,4}(t) < R) \cdot \\ &\quad \mathbb{P}(d_{4,5}(t) < R | d_{3,4}(t) < R) \, \mathbb{P}(d_{3,4}(t) < R) \\ &= \mathbb{P}(d_{1,2}(t) < R, d_{2,3}(t) < R, d_{3,4}(t) < R) \, \mathbb{P}(d_{4,5}(t) < R | d_{3,4}(t) < R) \end{aligned} \tag{23}$$

In the final expression of (23) we recognize the path availability of the 3-link path written in (22). Then, it should be clear that we can generalize, by induction, the expression of $A_{path}(\mathbf{d}_0, t)$ to any number n of hops.

In short, we can do better than the independence assumption among the links by accounting at least for the correlation between adjacent links. Indeed, the improved approximation requires to evaluate the joint probability:

$$\mathbb{P}(d_{i,i+1}(t) < R, d_{i+1,i+2}(t) < R)$$

In general, we can express the above joint probability in terms of the spatial pdf's $f_i(\mathbf{x}, t)$, $f_{i+1}(\mathbf{x}, t)$ and $f_{i+2}(\mathbf{x}, t)$ of nodes i, $i+1$, and $i+2$, respectively, at time t. It results:

$$\mathbb{P}(d_{i,i+1}(t) < R, d_{i+1,i+2}(t) < R) = \int_{\mathbf{x}_A} \int_{\mathbf{x}_B} \int_{\mathbf{x}_C} f_i(\mathbf{x}_A, t) f_{i+1}(\mathbf{x}_B, t) f_{i+2}(\mathbf{x}_C, t) \mathbb{1}_{||\mathbf{x}_A - \mathbf{x}_B|| < R} \mathbb{1}_{||\mathbf{x}_B - \mathbf{x}_C|| < R} \, d\mathbf{x}_A d\mathbf{x}_B d\mathbf{x}_C$$

In the case where nodes move according to a Brownian motion, the previous expression can be simplified, studying the joint evolution of the relative positions of nodes i and $i+2$ with respect to node $i+1$. By so doing, the dynamics of nodes i and $i+2$ as seen by node $i+1$ are still described by a Brownian motion, with non-null terms outside the diagonal of the corresponding covariance matrix, over an infinite four-dimensional domain.

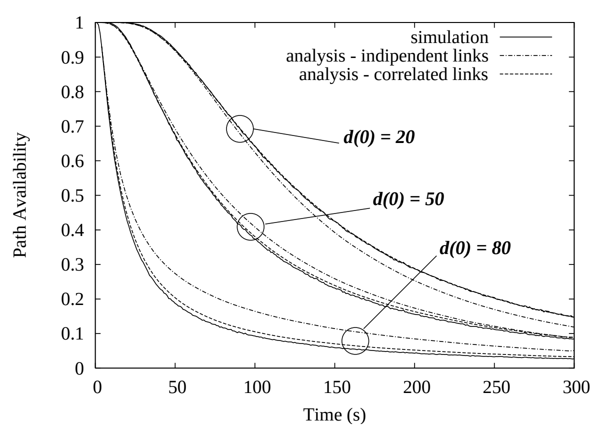

Fig. 6. Path availability under the Brownian motion, in case of 3 links and different values of initial hop lengths

[3]Note that, even if the events $d_{1,2}(t) < R$ and $d_{3,4}(t) < R$ are independent, they are not conditionally independent given the event $d_{2,3}(t) < R$

Figure 6 presents the availability of a path consisting of three hops, all having the same initial length, namely, 20, 50, and 80 m. We consider a Brownian motion with $\sigma^2 = 12$, and $R = 100$ m. Simulation results are compared with the approximation based on the link independence assumption, and with our improved approximation that partially accounts for the correlation between links. We observe that the improved approximation gets very close to the simulation results in all considered cases. The independence assumption slightly underestimates the path availability for an initial hop length of 20 m, whereas overestimates it in the other two cases. From extensive experiments, we have observed that, in general, the results obtained under the independence assumption become less accurate when the initial length of the hops is close to the transmission range (see the case $d(0) = 80$ m). However, it performs fairly well when the path availability is above, say, 0.5, i.e., in the cases of more practical interest for the applications.

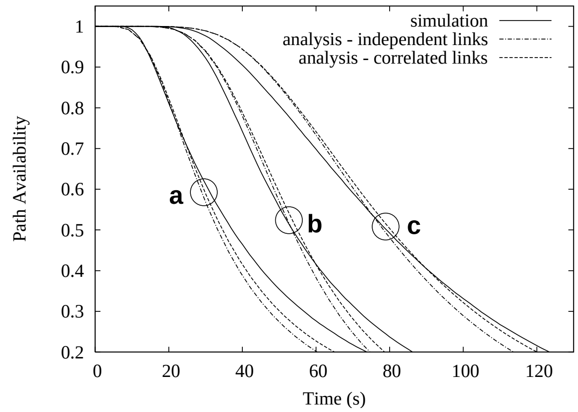

Fig. 7. Path availability in case of the RD model, for three different values of initial hop lengths

Finally, we present the results of a few experiments with the RD model. Note that in this case we have the combination of two approximations: one due to the normal approximation to compute the availability of a single link, the other due to our approximation (either independent or correlated links) to compute the availability of a multihop path. This time, we keep the initial length of each hop fixed to 20 m, and vary the parameters for the RD model considering three scenarios:

- **(a)** - 2 links, E[*move*] = E[*pause*] = 10 s, speed uniformly distributed in $[-5, 5]$ m/s.
- **(b)** - 5 links, E[*move*] = E[*pause*] = 30 s, speed uniformly distributed in $[-2, 2]$ m/s.
- **(c)** - 10 links, E[*move*] = E[*pause*] = 10 s, speed uniformly distributed in $[-2, 2]$ m/s.

Figure 7 presents the results obtained in these three scenarios, comparing simulation results with those obtained by our two approximations for the availability of a multihop path. We observe that both approximations are satisfactory when the path availability is not too low. Indeed, the use of the refined approximation accounting for link correlation does not provide significant improvements with respect to the one based on the independence assumption.

VII. Optimal Path Selection

We now show how the results we derived in the previous sections can be exploited to choose the optimal path between two nodes, in terms of route stability. In our discussion, we assume that the parameters of the underlying mobility model of the nodes are known a priori (e.g., derived through GPS).

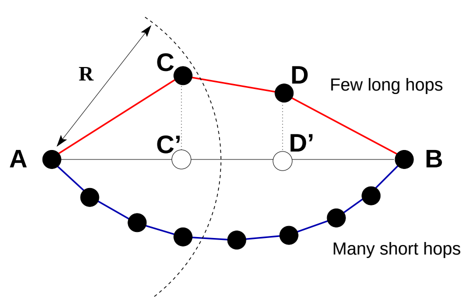

Fig. 8. Path selection between nodes A and B

Suppose node A wants to communicate with node B (see Figure 8), possibly using intermediate nodes as relays. To maximize the stability of the route in response to nodes' mobility, one can think of two different strategies: (i) a few long hops; (ii) many short hops. On the one hand, considering that the entire path fails if just a single link fails, and nodes move independently of each other, it seems better to minimize the number of hops. On the other hand, short links are much more stable than long links. These preliminary considerations suggest that there may be an optimal choice in between the extreme solutions of using few long hops (of length close to the nodes transmission range), or many short hops (in the limit, an infinite number of infinitesimal hops). Here we show some results that confirm such an intuition.

To shed some light onto the problem, let us assume that the density of the network nodes is so large that we can virtually select any point in the area to act as relay between nodes A and B. We choose as selection criterion the metric of path availability, as defined in Section III, because it allows simpler closed form expressions. Moreover, to proceed analytically, we rely on the independence assumption among links to compute the availability of the entire path (see Section VI). The conclusions drawn under these simplifications will be checked against simulation in a practical example presented at the end of the section.

First, we establish some structural properties of the link availability $A_{link}(d_{A,B}(0), t)$ that hold for a general class of mobility models. To simplify the notation, let $d_0 = d_{A,B}(0)$ be the initial distance between the two nodes forming the link. Let $a(z,t)$ be the pdf of the absolute distance z reached by a node at time t, from its initial position. We only require $a(z,t)$ to be a decreasing function of z (for any t). In particular, notice that this is true for both the Brownian motion and the RD model.

Lemma 1: The link availability $A_{link}(d_0, t)$, at any time t, is a decreasing function of d_0, provided that $a(z,t)$ is a decreasing function of z for any t.

Proof: Consider the relative motion of one node with respect to the other. Let $f(z,t)$ be the pdf of the distance reached by this node from its initial location. It results that $f(z,t)$ is also a decreasing function of z. Let $F(z,t)$ be the corresponding cumulative function. In the unidimensional case, we can write

$$A_{link}(d_0, t) \quad = \quad F(R - d_0, t) + F(R + d_0, t) - 1$$

If we compute the derivative of the above expression with respect to d_0 we obtain

$$\frac{\partial A_{link}(d_0, t)}{\partial d_0} = -f(R - d_0, t) + f(R + d_0, t) \tag{24}$$

which is negative because $f(R + d_0, t) < f(R - d_0, t)$. Hence $A_{link}(d_0, t)$ is a decreasing function of d_0.

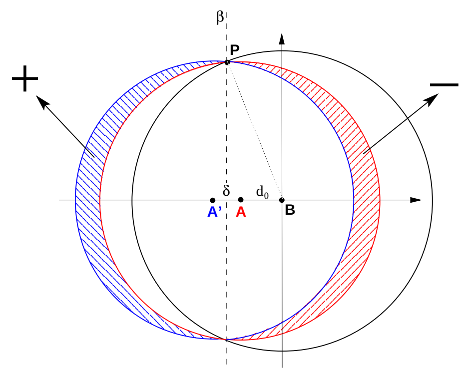

Fig. 9. Illustration of why the path availability is a decreasing function of d_0 in the bidimensional case

In the bidimensional case, the same property can be verified with the help of the diagram in Figure 9. Consider an arbitrary value d_0 of the initial distance between nodes A and B, $0 \leq d_0 < R$. In the coordinates system with origin at point B, the path availability $A_{link}(d_0, t)$ is obtained integrating $f(z,t)$ in the disc of radius R centered at point A. Now let us increase d_0 by an infinitesimal amount δ. The value of $A_{link}(d_0 + \delta, t)$ is obtained integrating $f(z,t)$ in the disc of radius R centered at point A' (see Figure 9), and we need to show that $A_{link}(d_0 + \delta, t) < A_{link}(d_0, t)$. When we move from d_0 to $d_0 + \delta$, the domain of integration is reduced by the area marked with the 'minus' sign, while it is increased by the area marked with the 'plus' sign, which is the symmetric of the 'minus' area with respect to line β. Now, consider the circle centered at point B with radius $\overline{BP}$, where P is the point marked on Figure 9. From simple geometric considerations, it results that the 'minus' area is entirely contained within this circle, whereas the 'plus' area is entirely outside the same circle. Since $f(z,t)$ is a decreasing function of d, the integral of $f(z,t)$ over the 'plus' area is less than the integral of $f(z,t)$ over the 'minus' area, and we can conclude that $A_{link}(d_0 + \delta, t) < A_{link}(d_0, t)$. ■

Proposition 1: The optimal path between two nodes A and B lies on the segment connecting the two nodes, provided that $a(z,t)$ is a decreasing function of z for any t.

Proof: By contradiction, suppose the optimal path is outside the segment $\overline{AB}$. Then, we could build another feasible path through the projections of the relay nodes on segment $\overline{AB}$ (in Figure 8, path $ACDB$ can be replaced by path $AC'D'B$), obtaining a path in which all hops are shorter than or equal to the hops of the original path. Based on (21) and the previous lemma, the constructed path has higher availability than the original path. ■

Proposition 2: For a given number of hops n, the optimal path has all hops of equal length, provided that $A_{link}(d_0, t)$ is a log-concave function of d_0.

Proof: Let L be the initial distance between A and B, and recall that $\mathbf{d_0} = \{d_{i,i+1}(0)\}$ is the vector of initial hop lengths $d_{i,i+1}(0)$, $1 \leq i \leq n$. Having fixed n, the optimal path is the solution of the optimization problem

$$\begin{aligned} \max \quad & \prod_{i=1}^{n} A_{link}(d_{i,i+1}(0), t) \\ \text{s.t.} \quad & \sum_{i=1}^{n} d_{i,i+1}(0) = L \\ & d_{i,i+1}(0) \geq 0, \ 1 \leq i \leq n \end{aligned} \tag{25}$$

where we have used the property that the optimal path must lie on the segment $\overline{AB}$ of length L. Taking the logarithm of the objective function, we can alternatively find the maximum of $\sum_{i=1}^{n} \log\left[A_{link}(d_{i,i+1}(0), t)\right]$. Therefore, if the link availability is a log-concave function of d_0, we obtain a convex optimization problem over a convex set in $\mathbb{R}^n$, which admits a unique global optimum. Setting to zero the gradient of the Lagrangian function:

$$\mathcal{L}(\mathbf{d}, \lambda) = \sum_{i=1}^{n} \log\left[A_{link}(d_{i,i+1}(0), t)\right] - \lambda \left(\sum_{i=1}^{n} d_{i,i+1}(0) - L \right)$$

we obtain the solution in which $d_{i,i+1}(0) = L/n$, $\forall i$. ■

Finding the conditions under which $A_{link}(d_0, t)$ is a log-concave function of d_0 is a difficult task. In the special case of a unidimensional mobility model, the only requirement is that $a(z, t)$ is a decreasing function of z. Indeed, we can derive (24) with respect to d_0, yielding:

$$\frac{\partial^2 A_{link}(d_0, t)}{\partial d_0^2} = f'(R - d_0, t) + f'(R + d_0, t) \tag{26}$$

which is negative if $A_{link}(d_0, t)$ is a decreasing function of d_0. In this case $A_{link}(d_0, t)$ would be a concave function of d_0, thus also log-concave. In the bidimensional case the analysis is more difficult. For example, $A_{link}(d_0, t)$ for the Brownian motion is not always concave. However we have verified (numerically) that it is log-concave for any choice of parameters, and for any t.

Next, let us consider the problem of finding the optimal number of hops n. This is a discrete optimization problem; to proceed analytically, we relax n to a continuous quantity x, and look for the maximum of $A_{link}(L/x, t)^x$ with respect to x. In case of a uni- or bidimensional Brownian motion, we can find some useful approximations for the optimal value of x, which we denote by x^*.

In particular, let us consider a unidimensional Brownian motion with infinitesimal variance σ^2 for which the expression of the link availability can be found in (9) in Section IV. We notice that reasonable values of link availability (say larger than 0.5) require $2\sigma^2 t$ to be small with respect to R^2. When this is the case, $A_{link}(d_0, t)$ can be made close to 1 by setting the hop length L/x much smaller than R. Thus we can assume $2\sigma^2 t \ll R^2$, $L \ll Rx$. It follows that in (9) we can approximate the Q function by $Q(y) \approx \frac{1}{\sqrt{2\pi} y} e^{-y^2/2}$, valid for $y > 1$ (true in our case). By considering that: $(1 - y)^n \approx 1 - ny$ when y is small (as in our case), after some calculations we can reduce to find the minimum of:

$$\frac{e^{-\frac{R^2 x^2 + L^2}{2\sigma^2 t x^2}}}{\frac{R^2 x^2 - L^2}{2\sigma^2 t x^2}} \left[\frac{Rx}{\sqrt{2\sigma^2 t}} \cosh\left(\frac{RL}{2\sigma^2 t x}\right) + \frac{L}{\sqrt{2\sigma^2 t}} \sinh\left(\frac{RL}{2\sigma^2 t x}\right) \right]$$

Neglecting L^2 with respect to $R^2 x^2$, we are left to minimizing the term in square brackets in the above expression. Approximating $\cosh(y) \approx 1 + \frac{1}{2} y^2$ and $\sinh(y) \approx y$, we obtain the minimum at $x^* = \frac{L}{\sqrt{2\sigma^2 t}} \sqrt{1 + \frac{R^2}{4\sigma^2 t}}$. Finally, considering that the number of hops must be an integer number and that it cannot be smaller than the minimum possible number of hops $\lceil L/R \rceil$, we can approximate the optimal number of hops in the unidimensional Brownian motion as:

$$n^*_{1-\mathrm{dim}} = \max\left(\lceil L/R \rceil, \left\lfloor \frac{L}{\sqrt{2\sigma^2 t}} \sqrt{1 + \frac{R^2}{4\sigma^2 t}} \right\rfloor \right) \tag{27}$$

A similar approximation can be carried out for the bidimensional Brownian motion, starting from (8). After some tedious computations, we have found the following approximation,

$$n^*_{2-\mathrm{dim}} = \max\left(\lceil L/R \rceil, \left\lfloor \frac{L}{\sqrt{2\sigma^2 t}} \sqrt{1 + \frac{R^2}{4\sigma^2 t}} - \frac{L}{2\sqrt{2\sigma^2 t}} \right\rfloor \right) \tag{28}$$

which turns out to be very good.

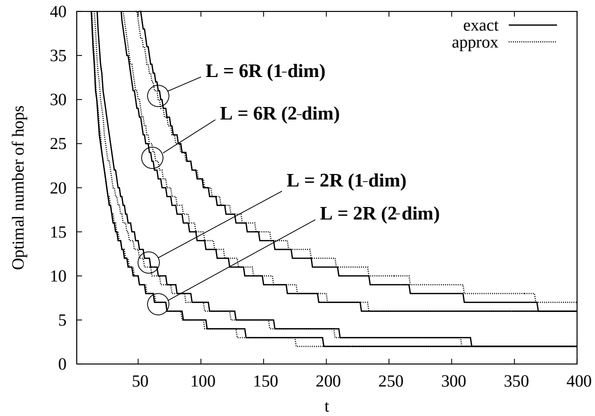

Fig. 10. Approximations and actual values of the optimum number of hops for the unidimensional and the bidimensional Brownian motion as a function of time

In Figure 10 we show the accuracy of our approximations of the optimum number of hops for a uni- and bidimensional Brownian motion with $\sigma^2 = 12$, in case of $R = 100$ m and $L = 2R$, $6R$. We compare the results provided by (27) and (28) with the actual optimum obtained after exploring numerically all values of n.

As predicted by our approximations, the optimum number of hops is proportional to L, the initial distance between A and B; for the same value of L, it is smaller in the bidimensional case than in the unidimensional case. Moreover, the optimum number of hops decreases with the passing of time. In particular, for large t (i.e., large $\sigma^2 t$) it approaches the minimum number of hops $\lceil L/R \rceil$.

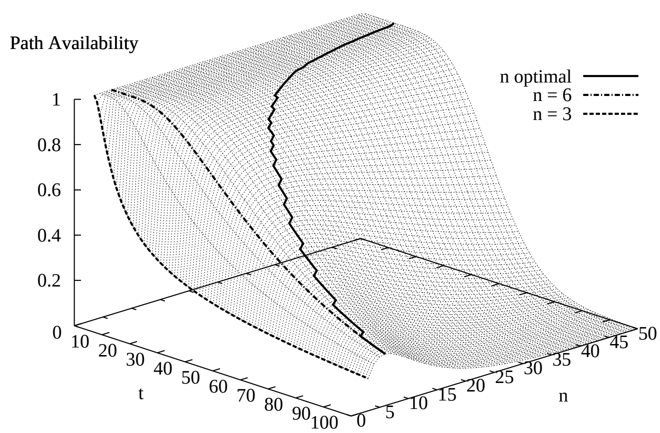

Fig. 11. Path availability as a function of t and n, in case of a bidimensional brownian motion with $L = 2.5\ R$

Note that, when $t \rightarrow 0$ ($2\sigma^2 t \rightarrow 0$), surprisingly the optimal number of hops tends to infinity. However, this is more a theoretical curiousness than something of practical interest. Indeed, when $2\sigma^2 t$ is very small, the path is available with probability close to 1 for any feasible number of hops. This is shown in Figure 11, where we report the path availability as a function of n and t, in the case of a bidimensional Brownian motion with $R = 100$ m and $L = 2.5\ R$. On this surface, we have plotted three curves obtained using (21): one corresponding to the minimum number of hops ($n = 3$), one with twice the minimum ($n = 6$) and the one corresponding to the actual optimum number of hops. We observe that the value of the path availability obtained when the optimum number of hops gets large (above 30) is practically granted also for much smaller values of n.

We also notice that a significant improvement can be achieved by just doubling the number of hops with respect to the minimum (i.e., passing from 3 to 6). To better show this fact, in Figure 12 we have plotted a few cuts of the surface in Figure 11, at different time instants. We observe that much of the gain can be achieved without increasing n too far beyond the minimum number of hops. Values of n around 2-3 times the minimum can dramatically improve the path availability for any value of t, up to the time at which the path availability becomes very small (below 0.2).

Similar conclusions can be drawn under the RD mobility model. As an example, Figure 13 reports the path availability

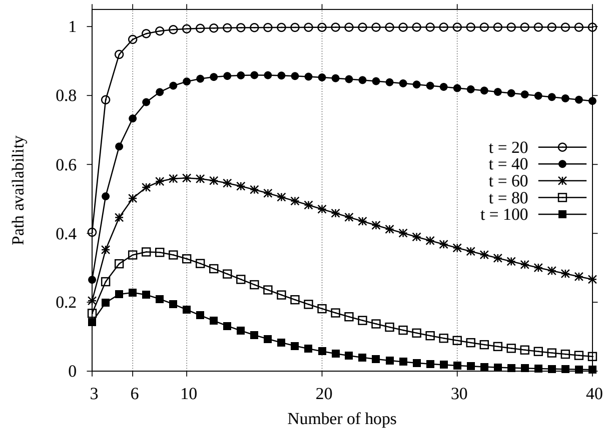

Fig. 12. Path availability at different time instants, as a function of n. Bidimensional Brownian motion with $L = 2.5\ R$

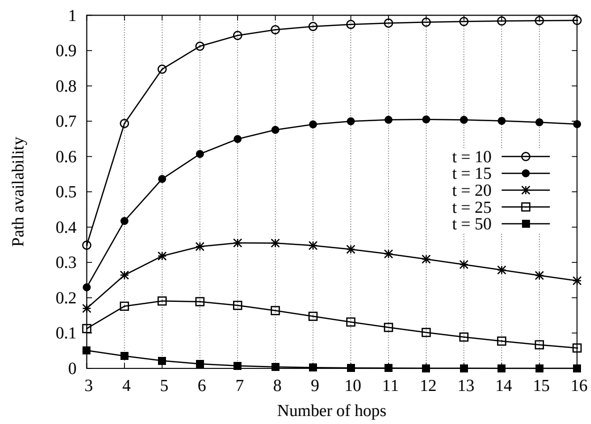

Fig. 13. Path availability at different time instants, as a function of n; bidimensional RD model with $L = 2.5\ R$

obtained using approximation (20) in the same scenario of Figure 12. This time we use an RD model in which *move* and *pause* times have an average of 10 s ($\lambda = \mu = 1/10$), while the speed is uniformly distributed in $[-5, 5]$ m/s. Again, we observe that the optimal number of hops decreases with time, and that significant improvements of the path availability can be obtained using a number of hops a few times larger than the minimum.

In this scenario, we have also run some simulations to check the validity of our analytical results. Figure 14 reports the path availability as a function of time obtained by simulation for three different values of n, namely 3,5,12. As expected, the path achieving the best performance varies over time. Up to about $t = 20$ s, the 12-hops path provides the best availability, significantly outperforming the minimum-hop path of $n = 3$. Between 20 and 50 s, the optimal path is the one with $n = 5$. Only after $t = 50$ s the minimum hop path becomes the best one, but at this point the path availability has already dropped to very low values. These results closely agree with the analytical predictions reported in Figure 13.

VIII. Conclusions

We studied the duration and availability of routing paths in MANETs – a fundamental issue to provide reliable routes and short route disruption times. We first focused on a bidimensional Brownian motion since (i) it is simple and analytically tractable; (ii) it provides insights on some system behaviors that can be exploited under different (more realistic) mobility models. The latter observation is based on the fact that the total movement of a node with respect to its initial position is determined by the sum of its elementary movements, for which the central limit theorem can be invoked to claim that after some time the node spatial distribution tends to become a normal distribution. In particular, we considered the Random Direction mobility model and showed that some results obtained for the Brownian motion come in handy in this case. Finally, we used our results on path duration and availability to determine the optimal path in terms of route stability, under both the Brownian and the Random Direction model. We showed some properties of the optimal path, and we provided an approximate expression for the optimal number of hops.

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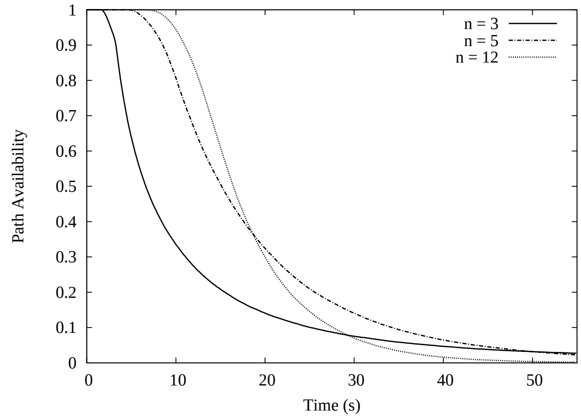

Fig. 14. Path availability as a function of time according to simulation, for different number of hops; bidimensional RD model with $L = 2.5\ R$

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Appendix

Here we show how (16) and (17) can be manipulated to get some important insights on the mobile dynamics, and obtain a different expression for $M(\mathbf{k}, s)$ and $p(\mathbf{k}, s)$ which can be easily numerically antitransformed.

Let us define $I(\mathbf{k}, s)$ as:

$$I(\mathbf{k}, s) = \int \frac{\lambda \mu f_V(\mathbf{v})}{(s+\lambda)(s+\mu+j\mathbf{k}\cdot\mathbf{v})} d\mathbf{v}$$

Observe that, by construction, for every $\mathbf{k}$ or s the fact that $|I(\mathbf{k}, s)| \leq 1$, and $|I(\mathbf{k}, s)| = 1$ entails $\mathbf{k} = 0$, $s = 0$; thus for $\mathbf{k} \neq 0$ or $s \neq 0$:

$$\frac{1}{1 - I(\mathbf{k}, s)} = \sum_{i=0}^{\infty} I^i(\mathbf{k}, s)$$

It follows that $M(\mathbf{k},s)$ can be rewritten as:

$$M(\mathbf{k},s)=\left(p_0(\mathbf{k})\frac{I(\mathbf{k},\mathbf{v})}{\mu}+\int\frac{m_0(\mathbf{k},\mathbf{v})}{s+\mu+j\mathbf{k}\cdot\mathbf{v}}d\mathbf{v}\right)\sum_{i=0}^{\infty}I^i(\mathbf{k},s) \tag{29}$$

which allows us to provide a physical interpretation for $I(\mathbf{k},s)$ as the Green's function related to the first cycle evolution of the system dynamics for a node that at time $t=0$ is in *pause* phase at $\mathbf{x}=0$. $I(\mathbf{k},s)$ can be antitransformed, obtaining:

$$I(\mathbf{x},t)=\lambda\mu\Big[e^{-\lambda t}\Big(\int_{0_+}^{t}\frac{f_V(\mathbf{x}/\tau)}{\tau}e^{-(\mu-\lambda)\tau}d\tau\Big)\mathbb{1}_{||x||>0}+\delta(\mathbf{x})e^{-\lambda t}\Big]$$

which provides the spatial distribution at time t for a node that is ending its first cycle. For finite t, $M(\mathbf{x},t)$ can be efficiently evaluated by directly antitransforming (29) after an opportune truncation of the series there in.