Optimizing the Topology of Bluetooth Wireless Personal Area Networks

Marco Ajmone Marsan, Carla F. Chiasserini, Antonio Nucci, Giuliana Carello, Luigi De Giovanni

***Abstract*— In this paper, we address the problem of determining an optimal topology for Bluetooth Wireless Personal Area Networks (BT-WPANs). In BT-WPANs, multiple communication channels are available, thanks to the use of a frequency hopping technique. The way network nodes are grouped to share the same channel, and which nodes are selected to bridge traffic from a channel to another, has a significant impact on the capacity and the throughput of the system, as well as the nodes' battery lifetime. The determination of an optimal topology is thus extremely important; nevertheless, to the best of our knowledge, this problem is tackled here for the first time.**

Our optimization approach is based on a model derived from constraints that are specific to the BT-WPAN technology, but the level of abstraction of the model is such that it can be related to the more general field of ad hoc networking. By using a *min-max* formulation, we find the optimal topology that provides full network connectivity, fulfills the traffic requirements and the constraints posed by the system specification, and minimizes the traffic load of the most congested node in the network, or equivalently its energy consumption. Results show that a topology optimized for some traffic requirements is also remarkably robust to changes in the traffic pattern. Due to the problem complexity, the optimal solution is attained in a centralized manner. Although this implies severe limitations, a centralized solution can be applied whenever a network coordinator is elected, and provides a useful term of comparison for any distributed heuristics.

I. INTRODUCTION

WIRELESS Personal Area Networks (WPANs) is a new wireless technology, which provides short-range connectivity between battery-operated portable radio devices, such as mobile phones, headsets and personal digital assistants. WPANs are intended to operate at the 2.4 GHz ISM (Industrial, Scientific, Medical) band, where no license is required, using a FHSS (Frequency Hopping Spread Spectrum) technique. This technology is based on the Bluetooth specification [1] and will become an IEEE standard under the denomination of 802.15 WPANs [2].

Bluetooth WPANs (BT-WPANs) are typically used to turn battery-operated stand-alone devices located in the range of about 10 m into networked equipment. The network nodes are organized into *piconets*, each of them composed of one *master* device and up to seven active *slaves*, which are allowed to communicate with the master only. Each piconet uses a different frequency hopping sequence derived from the master address. Master and slaves send and receive traffic alternatively, so as to provide full-duplex connections, and slaves are entitled to transmit only when polled by the master. It is intuitive that the master is subject to higher traffic load, and thus higher energy consumption, relative to slaves.

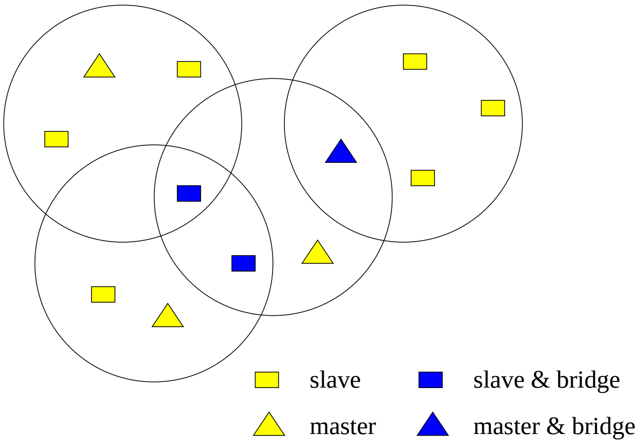

Fig. 1. BT-WPAN topology.

Figure 1 shows an example of BT-WPAN topology, where overlapping piconets are deployed. Being a master or a slave is only a logical state for nodes. A unit can participate in two or more overlying piconets, although it can be master in one piconet only. A master or a slave involved in the activity of more than one piconets can act as a *bridge*, allowing piconets to form a larger network, a so-called *scatternet*. Because of the use of different hopping sequences, a bridge cannot be active in more than one piconet at a time; thus, bridges have to switch between piconets on a time division basis, and, while switching, they must re-synchronize with the current piconet. This implies a significant overhead that may severely effect the system performance.

One of the most challenging problems in deploying a BT-WPAN consists in forming a scatternet that meets the constraints posed by the system specifications and the traffic requirements. The way nodes are grouped into piconets, and which nodes are selected as masters or bridges, has a significant impact on the capacity and the throughput of the system, as well as on the nodes' battery lifetime – a crucial factor in network connectivity [3], [4], [5]. Knowing which topology optimizes the BT-WPAN performance is therefore of fundamental importance.

So far, little research activity was devoted to algorithms for the generation of efficient topologies for BT-WPANs, and former work dealing with more general wireless ad hoc systems

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is not applicable. For instance, approaches based on the nodes' position, as the one proposed in [6], are suitable for networks that use a single communication channel, such as 802.11 wireless LANs, but cannot be applied to BT-WPANs where multiple channels are made available by the FH scheme [4]. Algorithms proposed in the context of multihop wireless networks, that control the topology by varying the nodes' transmit power [3], [7], [8], are not applicable as well. Indeed, BT-WPANs devices use a very low output power (namely, equal to 1 mW) and typically do not perform power control[1].

The issue of determining an optimal topology specifically for BT-WPANs is discussed in [4] but is not actually addressed there. The first attempt at finding a solution to the problem is represented by the work in [5]. Due to the problem complexity, the authors adopt a statistical approach, i.e., they generate random topologies and study the effect of the topology parameters on the system performance, deriving some useful insights: the bridging overhead and the number of links established between nodes are found to have a major impact on throughput, while slightly less important is the number of piconets that are created.

In this paper, we solve the problem of forming an optimal BT-WPAN topology that minimizes the traffic load of the most congested node in the network, or equivalently its energy consumption, and is such that:

1) full network connectivity is guaranteed;
2) system specifications are met;
3) the traffic requirements are fulfilled;
4) specific restrictions are met, that may exist on the role that some nodes can play.

To the best of our knowledge, this problem was not solved so far.

We provide a *min-max* formulation of the optimization problem, and we solve it in a centralized manner, since the problem complexity and the large number of parameters involved seem to prohibit a distributed approach. This is certainly a limitation of the proposed formulation. However, besides providing an upper bound to any topology derived through distributed heuristics, the centralized optimal solution can be applied whenever a network coordinator is elected. The work in [4] is an example of asynchronous distributed protocol that can be used to select a coordinator, which is in charge of determining the role of all nodes in the scatternet. Our scheme can be thought of as part of this protocol, and executed at the coordinator node to determine the optimal topology. Clearly, whenever there are significant modifications in the network composition, the coordinator needs to solve the problem for the new network configuration. However, our results show that the attained topology is surprisingly robust to changes in the traffic pattern.

Finally, although our model specifically addresses the issue of topology design in BT-WPANs, the level of abstraction is such that it can be applied to the more general case of ad hoc networks when multiple channel communications are available. The problem of determining which nodes share the same channel and which nodes have to bridge traffic from a channel to another can be posed in the same way, under the constraints of the specific physical layer and traffic requirements [4].

The remainder of this paper is organized as follows. Section II describes the network scenario and states the problem under study; Section III presents an Integer Linear Programming (ILP) formulation of the problem. Numerical results are presented and discussed in Section IV. Section V concludes the paper and points to some aspects that will be subject of future research.

II. PROBLEM STATEMENT

In BT-WPANs, connection establishment is a two-step procedure. By relaying on a universal frequency hopping sequence, first an *inquiry* protocol is used to let a node discover the units located in its proximity, then a *paging* protocol is used to establish the communication link between two units. The unit that initiates the procedure acts as the master of the connection, and the other unit as a slave, although roles can be exchanged later on. A master or slave can become a slave in another piconet by being paged by the master of the other piconet; also, a unit participating in one piconet can page the master or slave of another piconet. In this case, the master or slave unit acts as a bridge between the two overlapping piconets [1].

In this paper, we refer to a master as a node that is assigned to $p \geq 1$ piconets and is the master in one of them, to a slave as a slave node that is assigned to one piconet only, and to a bridge as a node that is assigned to $p \geq 2$ overlapping piconets and is a slave in all of them.

Being BT-WPAN technology intended for local connectivity of battery-driven equipment, it supports low-power implementations. The output power of BT-WPAN devices is limited to 1 mW and the entire radio tranceiver is designed for low power consumption. The major factor influencing the nodes' energy consumption becomes the amount of transmitted, received, and processed traffic rather than the distance between transmitter and receiver. Notice that masters and bridges are therefore the nodes that experience the highest energy consumption in the network.

Designing the BT-WPAN topology means determining the set of communication links between node pairs that are used to route traffic from each source to the corresponding destination node. Consider a set of network nodes; we look at the topology formation problem as one of optimizing a chosen performance metric under a given set of constraints. We select as performance metric to be minimized, the traffic load of the most congested node, or equivalently its energy consumption, and we require that the optimal BT-WPAN topology meets the following constraints.

1) **Full network connectivity.** There must be at least one path between any two nodes in the network. This implies that all the masters have to be connected to each other, either through master or bridge nodes; instead, slaves can communicate with any node in the network through the master they are connected with.
2) **System specification.** The number of nodes participating in a piconet cannot be greater than a given value, and the distance between each master-slave pair must be less than

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[1] BT-WPAN applications can be in the range of 10 or 100 m; however, the typical usage model is in the range of 10 m. Power control is only used by devices operating in the range of 100 m.

the maximum piconet radius. Also, a node can be master in one piconet only.

3) **System complexity.** In order to keep the network complexity small, the number of formed piconets is limited to a fixed value.
4) **Traffic requirement.** The network must support the desired source-destination connections.
5) **Constraints on the nodes' role**. Constraints on the role that some nodes can have in the network may exist. A node may need to act as either a slave or a master, depending on the application it is able to support, and on the nature of the device. For instance, nodes that are gateways to the fixed network should be chosen as masters.

III. ILP FORMULATION

In this section, first we introduce our notation and definitions, then we describe the mathematical formulation of the problem and the topology constraints. Finally, an example of how the optimization procedure works is given.

A. Notation and Definitions

The BT-WPAN is represented as $(\mathcal{N}, Z)$, where $\mathcal{N}$ is the set of network nodes and Z is the matrix containing the values of the distances between any two nodes $(i, j),\ i, j \in \mathcal{N}$. Let $\mathcal{S}$ be the set of traffic sources and $\mathcal{D}$ be the set of destination nodes. N, S and D denote the numbers of nodes, sources and destinations, respectively, while Z_{MAX} is the maximum radius of a piconet.

We define $\mathcal{C} = \{(s, d) : s \in \mathcal{S},\ d \in \mathcal{D}\}$ as the set of source-destination connections, and $C = |\mathcal{C}|$ as the total number of connections that have to be routed through the network. We assume that just one route is used for each source-destination pair. Let $T = \{t_{ij}\}$ be the traffic matrix indicating the information rate on each source-destination connection, normalized to the network capacity. For each traffic source $s,\ s \in \mathcal{S}$, we take the total average traffic rate, denoted by ρ_s, as an input parameter to the problem.

For each node $i,\ i \in \mathcal{N}$, three binary variables are defined: μ_i, β_i and σ_i, which are equal to 1 if node i is a master, a bridge or a slave, respectively, and are equal to 0 otherwise. We denote the maximum number of piconets by M_{MAX} and the maximum number of active nodes that can be assigned to a piconet by X_{MAX}. Since a piconet includes one and only one master, we refer to the generic piconet i through its master node. The set of nodes that are forced to be masters is indicated by $\mathcal{M}$, with $|\mathcal{M}| \leq M_{MAX}$; the set of nodes that are forced to be slaves is indicated by $\mathcal{V}$, with $|\mathcal{V}| \leq N - M_{MAX}$.

For each pair of nodes $(i, j),\ i, j \in \mathcal{N}$, we define the set of assignment variables, $\mathcal{X} = \{x_{ij}\}$, as follows

$$x_{ij} = \begin{cases} 1 & \text{if } j \text{ is assigned to master } i \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and the set of flow variables, $\mathcal{F} = \{f_{ij}\}$, as

$$\begin{array}{ll} f_{ij} > 0 & \text{if there exists any flow from } i \text{ to } j \text{ and } i \neq j \\ f_{ij} = 0 & \text{otherwise.} \end{array} \tag{2}$$

For each source-destination pair $(s, d) \in \mathcal{C}$, and for each pair of nodes $(i, j),\ i, j \in \mathcal{N}$, we introduce the following routing variables,

$$r_{ij}^{sd} = \begin{cases} 1 & \text{if connection } (s,d) \text{ is routed on arc } (i,j) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

so that the set $\mathcal{R} = \{r_{ij}^{sd}\}$ defines the connection path through the network for any connection in $\mathcal{C}$.

B. Optimization Problem

Consider a set of network nodes with distance matrix Z and a set of source-destination connections, $\mathcal{C}$, with traffic matrix T.

Given the set of routing variables $\mathcal{R}$, the traffic load of the generic node $i,\ i \in \mathcal{N}$, is defined as the sum of the incoming and outgoing traffic that i has to handle. For each connection $(s, d) \in \mathcal{C}$, we have that the load of node i is

$$L_i^{sd} = \sum_{k \in \mathcal{N}} r_{ki}^{sd} \rho_s\, t_{sd} + \sum_{j \in \mathcal{N}} r_{ij}^{sd} \rho_s\, t_{sd} \tag{4}$$

where the first term is the amount of traffic related to connection (s, d) that node i receives, and the second term represents the traffic that i has to forward to the next node in the connection path. By adding L_i^{sd} over all the connections in $\mathcal{C}$, we obtain the load of node i as

$$L_i = \sum_{(s,d) \in \mathcal{C}} L_i^{sd}\,. \tag{5}$$

We define as the network *bottleneck* the node that experiences the highest traffic load, i.e., whose traffic load, B, is

$$B = \max_{i \in \mathcal{N}} L_i\,. \tag{6}$$

Our objective is to select the network topology so as to obtain the optimal BT-WPAN topology, which minimizes the traffic load of the most congested node, i.e., its energy consumption, while guaranteeing the desired throughput. This is defined to be the optimization problem identified as $\mathbf{P}$:

$$\mathbf{P}: \quad \min_{\{r_{ij}^{sd} \in \mathcal{R}\}} B\left(T, \{\rho_s \in \mathcal{S}\}, \{r_{ij}^{sd} \in \mathcal{R}\}\right)$$

$$\text{subject to constraints on } \{x_{ij}\}, \{f_{ij}\}, \{r_{ij}^{sd}\}\,. \tag{7}$$

Constraints on assignment variables $\{x_{ij}\}$ are as follows.

$$\mu_i + \beta_i + \sigma_i = 1 \quad \forall\, i \in \mathcal{N} \tag{8}$$

$$\sum_{i \in \mathcal{N}} x_{ij} \leq \sigma_j + |\mathcal{N}| \cdot \beta_j + |\mathcal{N}| \cdot \mu_j \quad \forall\, j \in \mathcal{N} \tag{9}$$

$$\sum_{i \in \mathcal{N}} x_{ij} \geq 2 - \sigma_j - \mu_j \quad \forall\, j \in \mathcal{N} \tag{10}$$

$$x_{ii} = \mu_i \quad \forall\, i \in \mathcal{N} \tag{11}$$

$$x_{ij} \cdot z_{ij} \leq Z_{MAX} \cdot \mu_i \quad \forall\, i, j \in \mathcal{N} \tag{12}$$

$$\sum_{j \in \mathcal{N}} x_{ij} \leq X_{MAX} \cdot \mu_i \quad \forall\, i \in \mathcal{N} \tag{13}$$

$$2 + x_{ji} \geq \mu_i + \mu_j + x_{ij} \qquad \forall\, i, j \in \mathcal{N},\; i \neq j \tag{14}$$

$$x_{ik} + x_{jk} \leq 4 - \mu_i - \mu_j - x_{ij} \qquad \forall\, i, j, k \in \mathcal{N},\; i \neq j, k,\; j \neq k \tag{15}$$

$$\sum_{i \in \mathcal{N}} \mu_i \leq M_{\text{MAX}} \tag{16}$$

$$\sum_{i \in \mathcal{M}} \mu_i = |\mathcal{M}| \tag{17}$$

$$\sum_{i \in \mathcal{V}} \sigma_i = |\mathcal{V}|\,. \tag{18}$$

Constraint (8) ensures that a node is either a master, or a slave or a bridge. Constraint (9) ensures that a slave is assigned to one master at most; (10) ensures that a slave or a master are assigned to one piconet at least, while a bridge is assigned to two piconets at least; (11) forces a master to be assigned to itself. According to requirement 2 in Section II, inequality (12) forbids assigning a node to a master if their distance is greater than Z_{MAX}, and (13) limits the number of nodes assigned to a piconet to X_{MAX}. Given that nodes i and j are masters, (14) forces the assignment of i to j if j is assigned to i; (15) prevents cycles among sets of three nodes which are either masters or bridges: if master i is assigned to master j, node k cannot be assigned to both i and j. Notice that this constraint is not posed by the system specification, but was introduced in order to keep the complexity of the network topology small. Based on criteria 3 and 5, constraint (16) guarantees that the total number of masters is less than or equal to M_{MAX}, and (17) forces nodes in $\mathcal{M}$ to be masters. Constraint (18) forces nodes in set $\mathcal{V}$ to be slaves.

Next, in order to meet requirement 1 in Section II, we consider a graph connecting all the masters in the network. The constraints on the flow variables $\{f_{ij}\}$, guarantee that all masters are connected to each other either through master or bridge nodes. We take a master, denoted by μ_0, as the origin of the graph; every other master is a sink node. Notice that any master can be taken as origin of the graph and that the solution of problem $\mathbf{P}$ does not depend on which master node is chosen. We have

$$\sum_{j \in \mathcal{N}} f_{ij} - \sum_{j \in \mathcal{N}} f_{ji} = -\mu_i \qquad \forall\, i \in \mathcal{N},\; i \neq 0 \tag{19}$$

$$\sum_{j \in \mathcal{N}} f_{0j} - \sum_{j \in \mathcal{N}} f_{j0} = \sum_{k \in \mathcal{N}} \mu_k - \mu_0 \tag{20}$$

$$f_{ij} \leq M_{\text{MAX}} \cdot (x_{ij} + x_{ji}) \qquad \forall\, i, j \in \mathcal{N} \tag{21}$$

$$f_{ii} = 0 \qquad \forall\, i \in \mathcal{N}\,. \tag{22}$$

Constraints (19) and (20) guarantee flow conservation, i.e., they ensure that the flow originated in μ_0 reaches every master in the network. Inequality (21) ensures that there is no flow between node i and node j if neither i is assigned to j nor j is assigned to i. Constraint (22) forces the flow between a node and itself to be equal to 0.

Finally, given the source-destination connections that the network has to support, the constraints on the routing variables are as follows.

$$\sum_{j \in \mathcal{N}} r_{ij}^{sd} - \sum_{j \in \mathcal{N}} r_{ji}^{sd} = 1 \qquad \forall\, (s, d) \in \mathcal{C}, s \neq d,\; i = s \tag{23}$$

$$\sum_{j \in \mathcal{N}} r_{ij}^{sd} - \sum_{j \in \mathcal{N}} r_{ji}^{sd} = -1 \qquad \forall\, (s, d) \in \mathcal{C}, s \neq d,\; i = d \tag{24}$$

$$\sum_{j \in \mathcal{N}} r_{ij}^{sd} - \sum_{j \in \mathcal{N}} r_{ji}^{sd} = 0 \qquad \forall\, (s, d) \in \mathcal{C}, s \neq d,\; \forall\, i \in \mathcal{N}, i \neq s, d \tag{25}$$

$$r_{si}^{sd} \leq x_{is} + x_{si} \qquad \forall\, (s, d) \in \mathcal{C}, \forall\, i \in \mathcal{N} \tag{26}$$

$$r_{id}^{sd} \leq x_{id} + x_{di} \qquad \forall\, (s, d) \in \mathcal{C}, \forall\, i \in \mathcal{N} \tag{27}$$

$$r_{ij}^{sd} \leq f_{ij} + f_{ji} \qquad \forall\, (s, d) \in \mathcal{C}, \forall\, i, j \in \mathcal{N},\; i \neq s, j \neq d \tag{28}$$

$$\sum_{j \in \mathcal{N}} r_{ij}^{sd} \leq 1 \qquad \forall\, (s, d) \in \mathcal{C}, \forall\, i \in \mathcal{N} \tag{29}$$

$$r_{ij}^{sd} + r_{ji}^{sd} \leq 1 \qquad \forall\, (s, d) \in \mathcal{C}, \forall\, i, j \in \mathcal{N}\,. \tag{30}$$

Equations (23)-(25) are constraints on flow conservation and ensure that for each $(s, d) \in \mathcal{C}$ there is a route connecting s to d. Inequalities (26) and (27) allow a connection to be routed through edge (i, j) only if i and j communicate, i.e., either i is assigned to j, or j is assigned to i. A connection can be routed through a pair of nodes (i, j), which are masters or bridges, if edge (i, j) belongs to the graph connecting all the masters (28). Constraints (29) and (30) guarantee that loop-free routes are established.

C. Remarks

It can be shown that problem $\mathbf{P}$ is at least as complex as the Geometric Connected Dominating Set problem, which is proven to be NP-complete [9]; hence $\mathbf{P}$ is NP-complete.

By solving $\mathbf{P}$, we obtain the optimal BT-WPAN topology, which minimizes the traffic load of the most congested node, i.e., its energy consumption, while guaranteeing the desired throughput. Then, for any source-destination pair (s, d), we can verify whether an alternative route exists by using the following procedure. We fix variables σ_is, β_is, μ_is, and f_{ij}s to the values that they take in the solution of $\mathbf{P}$, while we set to 0 the variables r_{ij}^{sd} whose value is equal to 1 in the solution. By doing so, we generate a new problem, $\mathbf{P}'$, in which the solution obtained by solving $\mathbf{P}$ is forbidden. If $\mathbf{P}'$ is feasible, its solution provides a new route for (s, d); otherwise, it is proved that an alternative route for (s, d) does not exist.

Finally, we observe that we did not include in the model any constraint related to the capacity of the network nor of the single radio link; constraints related to the system capacity can be verified a posteriori. This is motivated by the fact that, even if a solution does not meet the capacity constraints, it suggests how much we should scale the traffic down in order to attain a feasible topology.

D. An Example

For the sake of clarity, here we present an instance of the optimization problem. We assume that network nodes are in-

TABLE I

INPUT PARAMETERS.

N	C	M_{MAX}	X_{MAX}	Z_{MAX}	$\mathcal{M}$	$\lvert\mathcal{V}\rvert$
20	15	4	8	$\frac{10\sqrt{2}}{3}$	$\{7, 17\}$	0

TABLE II

ROUTING OF NETWORK CONNECTIONS.

Connection	Traffic	Routes	No. hops
$(0, 12)$	0.291	$\{0, 16, 7, 12\}$	3
$(1, 14)$	0.066	$\{1, 0, 13, 14\}$	3
$(2, 17)$	0.764	$\{2, 13, 9, 17\}$	3
$(4, 0)$	0.081	$\{4, 13, 0\}$	2
$(6, 10)$	0.063	$\{6, 7, 16, 0, 13, 10\}$	5
$(7, 15)$	0.295	$\{7, 15\}$	1
$(8, 13)$	0.946	$\{8, 17, 9, 13\}$	3
$(11, 6)$	0.328	$\{11, 0, 16, 7, 6\}$	4
$(12, 7)$	0.077	$\{12, 7\}$	1
$(13, 2)$	0.872	$\{13, 2\}$	1
$(14, 5)$	0.548	$\{14, 13, 0, 16, 7, 5\}$	5
$(15, 9)$	0.518	$\{15, 7, 9\}$	2
$(17, 16)$	0.504	$\{17, 9, 13, 0, 16\}$	4
$(18, 19)$	0.100	$\{18, 7, 15, 17, 19\}$	4
$(19, 18)$	0.891	$\{19, 17, 15, 7, 18\}$	4

dependently, identically, and uniformly distributed in a $Q \times Q$ region with $Q = 10$ distance units, and the input parameters to the problem are as reported in Table I. Note that nodes 7 and 17 are forced to act as masters. The active source-destination connections and the associated traffic are listed in the first two columns of Table II.

The optimal topology attained by solving problem $\mathbf{P}$ is shown in Figure 2, where continuous lines represent the node-master assignments and dotted lines delimit the picocells' area. Columns 3-4 of Table II report for each network connection the selected route, along with the corresponding number of hops.

From the problem solution we have that, besides nodes 7 and 17, nodes 0 and 13 are chosen as masters. In fact, the optimization procedure selects a number of masters equal to M_{MAX}, so that the masters' traffic load is reduced. Masters 0 and 13 are directly connected to each other, while masters 7 and 17 have to be connected to other masters through bridge nodes because they are not located in overlapping areas.

Nodes 9, 15 and 16 are selected as bridges. Observe that bridge 9 would be sufficient to provide connectivity for the whole network; however, by electing also 15 and 16 as bridges, the load of bridge 9 and master 13 decreases. For instance, connection (11,6) is routed through nodes $\{0, 16, 7\}$ instead of being routed on $\{0, 13, 9, 7\}$, thus avoiding 13 and 9 to be overloaded.

Although in (6) the maximum traffic load is computed over all network nodes, the bottleneck node is likely to be either a master or a bridge. In this example, the bottleneck is master 7, giving the objective function a value of 0.596, as shown in Table III.

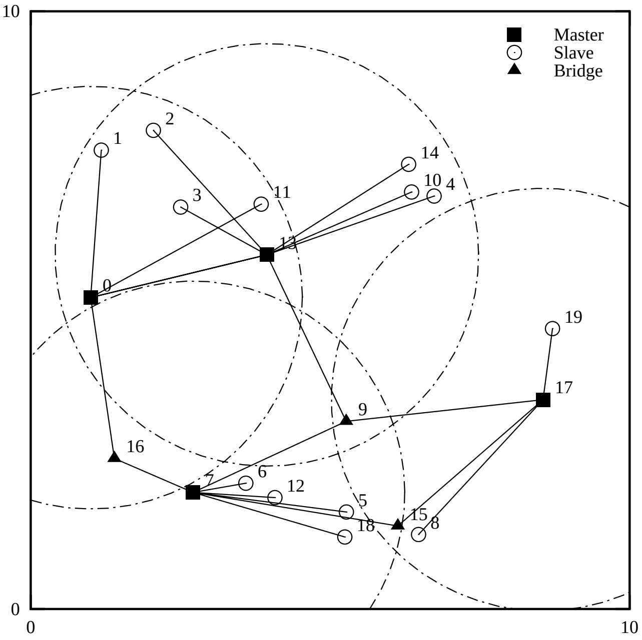

Fig. 2. Optimal topology obtained for the input parameters in Table I and the traffic demand in Table II.

TABLE III

TOTAL TRAFFIC THROUGH MASTERS AND BRIDGES.

Node	Total traffic
7	0.596
13	0.587
17	0.514
9	0.495
0	0.350
16	0.307
15	0.279

IV. NUMERICAL RESULTS

The numerical results presented in this section are derived by using the input parameters reported in Table I. We assume $\rho_s = \rho \; \forall \, s \in \mathcal{S}$, with ρ a varying parameter of the system. Plots are derived by averaging the results obtained from several runs, each of them corresponding to a different instance of the random variables of the system model. Each problem instance is solved by using the software tool CPLEX, which solves mixed integer problems by applying a branch and bound algorithm [10]. By using a Pentium II 266 MHz, the computation time per instance is equal to 7268.31 s on average and equal to 14086.74 s in the worst case.

Figure 3 shows the traffic load of the bottleneck node, B, as a function of the average source rate, ρ. For each value of ρ, results are plotted for two different cases. Results labeled in the plot by OPT are derived by assuming that for each problem instance the traffic matrix is known and optimizing the topology for such a traffic scenario. Results la-

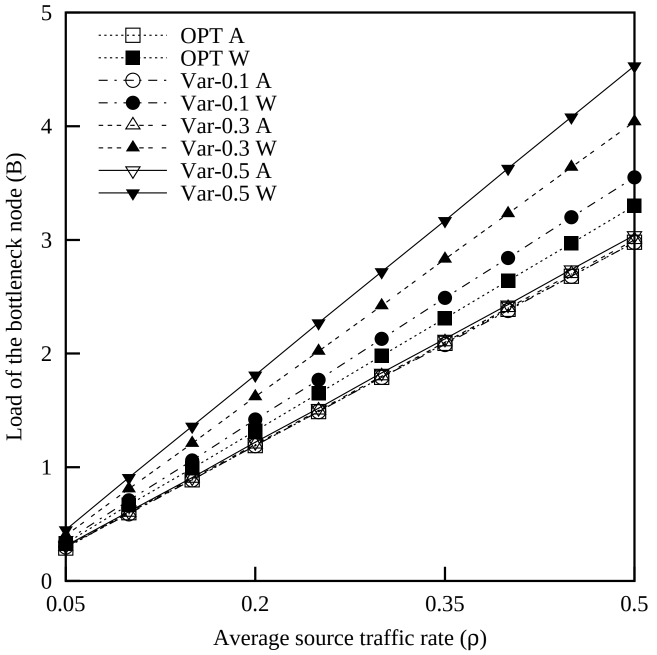

Fig. 3. Traffic load of the bottleneck node (B) vs. the average source rate (ρ). Curves labeled by OPT are derived for the optimal topology when the traffic matrix is known. Results labeled by Var-x ($x = 0.1, 0.3, 0.5$) are obtained for the same topology and source-destination connections, but introducing a variance equal to x in the traffic load. Labels A and W indicate the average and the worst case performance, respectively, over different instances.

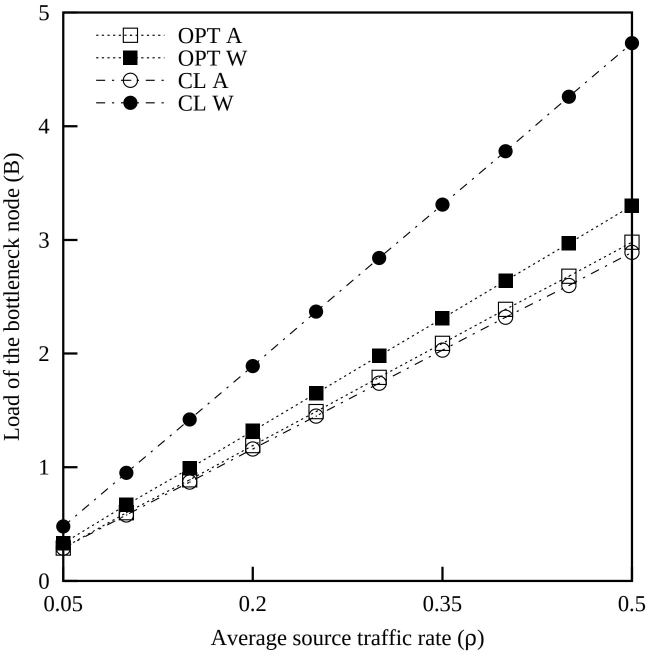

Fig. 4. Traffic load of the bottleneck node (B) vs. the average source rate (ρ). For each value of ρ, curves labeled by CL are obtained for a topology optimized for a particular instance of the traffic matrix and by varying both the source-destination connections and their traffic rates. Results are compared with the performance derived under the network conditions for which the topology has been optimized (labeled by OPT). Labels A and W indicate the average and the worst case performance, respectively, obtained from different runs.

beled by Var-x ($x = 0.1, 0.3, 0.5$) are obtained by maintaining for each instance the topology derived in the OPT case and the same source-destination connections, but introducing a variance equal to x in the traffic flowing over each source-destination connection. For each source-destination connection, the corresponding entry in the traffic matrix is an instance of a random variable uniformly distributed between 0 and 1. This enables us to test the topology generated for given traffic conditions in network scenarios where only the average connections' rates are known. Labels A and W denote the average and the worst case performance, respectively, that are obtained from the different runs.

Looking at Figure 3, we notice that when the average performance is considered, curves corresponding to any of the Var-x cases overlap the curves derived for the OPT case. This shows that the attained topology is surprisingly robust to changes in the connections rate. When the worst case performance is considered, the load of the bottleneck node increases with the uncertainty introduced in the traffic demand. However, the performance derived in the Var-x cases are still very close to that of the OPT case, especially for small values of ρ.

Figure 4 shows the traffic load of the bottleneck as a function of ρ, for a network scenario where only the average traffic load of the network is known (labeled in the plot by CL). The topology is optimized for a particular instance of the traffic matrix and the network performance is computed as the source-destination connections and their traffic rates are varied. Sources and destinations are randomly chosen among the network nodes. The results derived in the CL case are compared with those obtained in the OPT case. Again, labels A and W indicate the average and the worst case performance, respectively.

The curve representing the average performance of the CL case is still remarkably close to the one referring to the OPT case; instead, a larger gap can be observed between curves representing the worst performance in the two cases. This was expected, since in this case the optimization is performed with very limited information about the traffic carried by the network.

Next, we introduce as performance metric of the BT-WPAN, the nodes' residual energy at the time instant when the bottleneck runs out of energy. We denote the residual energy of the generic network node by e_r. Let us assume that all network nodes have the same initial amount of available energy, equal to 1. We consider the *network lifetime* to be the period from the time instant when the network starts functioning to the time instant when the first node runs out of energy, as first defined in [11]. Since the main factor to power consumption in BT-WPANs is the amount of traffic received, transmitted, and processed by the nodes, we can assume that the BT-WPAN lifetime is given by $1/B$, with B being the bottleneck load. Hence, the total energy consumed by the generic node in the time span corresponding to the network lifetime is given by the ratio of its traffic load to the load of the bottleneck node. The node's residual energy is computed as the difference between the energy initially available at the node and the amount of energy it has consumed.

Table IV presents the mean and variance of the residual energy and traffic load of the master and bridge nodes. Indeed, these nodes are the ones that determine the network perfor-

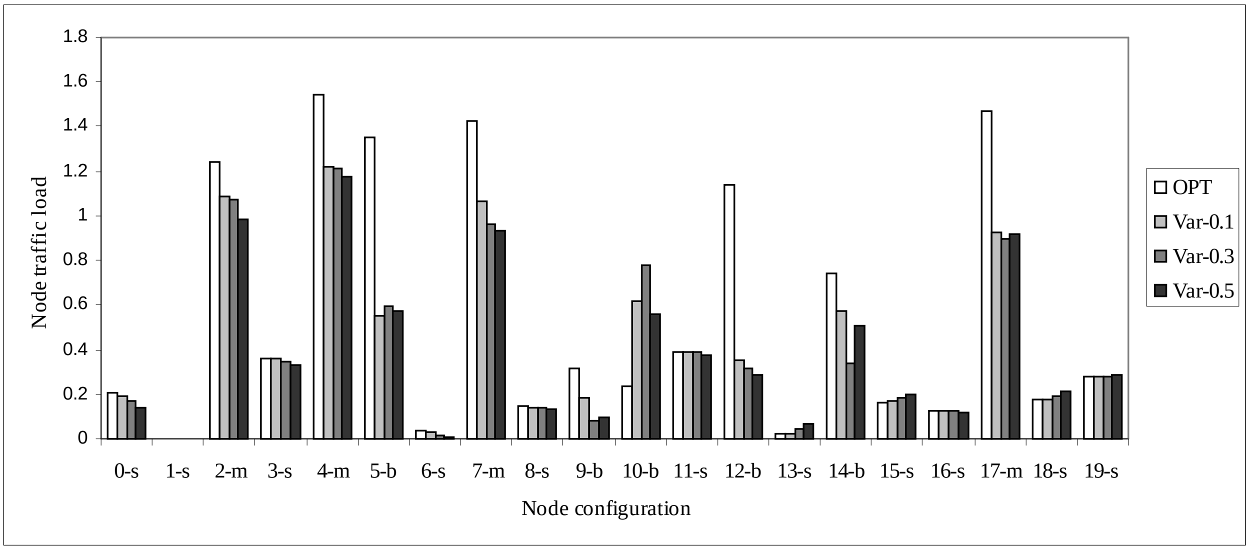

Fig. 5. Traffic load of the network nodes for a topology attained as a solution to a particular instance of the optimization problem. The role of the nodes in the topology is indicated as: m=master, b=bridge, s=slave. Results labeled by OPT refer to the network scenario for which the topology has been optimized. Results labeled by Var-x refer to the case where an uncertainty equal to x has been introduced in the traffic demand.

TABLE IV

Residual energy and traffic load of master and bridge nodes for $\rho = 0.3$. The source-destination connections are known; x is the variance introduced in the traffic demand.

x	Ave. e_r	Var. e_r	Ave. Load	Var. Load
0	0.358	0.103	0.990	0.246
0.1	0.451	0.105	0.668	0.181
0.3	0.455	0.122	0.654	0.170
0.5	0.477	0.123	0.609	0.153

mance, since they process more data, and hence consume more energy than slaves.

The results are derived for $\rho = 0.3$ for both the OPT and Var-x cases, as described above (in the table, $x = 0$ corresponds to the OPT case).

The average value of the residual energy is minimum in the OPT case and increases with x. In fact, the closer the network scenario is to the one for which the topology has been optimized, the longer is the network lifetime and the less is the residual energy at the nodes. On the contrary, the variance in residual energy does not significantly change as x varies.

Looking at the mean and variance of the nodes' traffic load, we notice that it is maximum in the OPT case and decreases as x increases. Thus, minimizing the load of the bottleneck does not correspond to minimizing the average value or the variance of the masters and bridges' load. This suggests that, depending on the application supported by the BT-WPAN and the nodes' physical characteristics, a different objective function which leads to a better balance of the network traffic load might be desirable.

Finally, Figure 5 shows the traffic load of all nodes for the topology attained by solving a particular instance of the optimization problem. Masters, bridges, and slaves are denoted by m, b, and s, respectively. The network bottleneck is node 4. The results derived for the network scenario for which the topology has been attained (labeled in the figure by OPT) are compared with the results obtained when a variance equal to x is introduced in the traffic flowing over each source-destination connection (labeled in the figure by Var-x, $x = 0.1, 0.3, 0.5$). Notice that in this particular instance, the connections' rates generated for the Var-x cases are lower than those in the traffic matrix of the OPT case; thus, the load of the bottleneck node is higher in the OPT case than in the Var-x cases. We also observe that the master nodes have the highest value of traffic load, and the difference between the load of masters and slaves is about one order of magnitude.

V. Conclusions and Future Work

In this paper, we tackled the problem of determining an optimal topology for Bluetooth Wireless Personal Area Networks. For this problem we provided an Integer Linear Programming formulation, an example of solution, and some numerical results.

The construction of an optimal topology for Bluetooth Networks is particularly important, since the network lifetime critically depends on the amount of energy consumed by battery-operated terminals, and the latter is determined by the amount of information handled by each device, hence by the network topology.

Results show that optimized topologies can be quite robust to changes in the traffic pattern. This is a very interesting feature, since traffic forecasts used for the topology optimization are bound to be only partly reliable.

The main limitation of our approach lies in the centralized nature of the optimization algorithm. However, it must be observed that a centralized solution can be applied whenever a network coordinator is elected; moreover, the availability of the results of a centralized optimization provides a useful term of comparison for any distributed heuristics. Nevertheless, the obvious next step in our research in this field will be the identification of heuristic approaches for the construction of good topologies in a distributed fashion.

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